Time-of-Flight Polarimeter

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This report describes the implementation of a Time-of-Flight (ToF) transient imaging polarimeter that produces an image with the ToF profile of a scene including measurement of polarisation properties. Recent developments in ToF systems have made it affordable to produce transient images of a scene, capturing the optical impulse response. This describes light propagation through the scene during short time intervals. The aim of the project was to develop a polarimeter based on recent ToF systems to generate transient images with measurement of polarization properties. The produced image with polarisation properties will gather valuable information that is not visible with intensity only ToF images. This enhances transient imaging performance through greater contrast and distinction. The instrument provides polarisation state control of the illuminator and receiver to obtain polarised ToF transient images. Experimental results show the ToF polarimeter requires increased focusing to obtain accurate ToF Mueller matrices.

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I. Introduction

By illuminating a scene by a controlled source and detecting the backscattered photons, important information of the scene is captured. This technique is utilised in most active imaging systems, providing more information about the scene, when the object or secondary source is outside our control. Obtained images through active imaging provided parameters different than just reflectivity, providing new information within the image such as range of objects, detection of moving objects, material distinction of objects, etc.

The polarisation state of light contains important information of a scene complementary to details provided by the intensity and wavelength of light. Through imaging polarimeters, the polarization state of light is measured incident upon a camera. The output of the instrument is a set of Mueller matrix element images. Greater information about the observed objects and scenes is provided through various derived polarimetric parameters. The advantages of polarimetry over conventional images includes advancement in distinguishing between objects and the natural background with the same reflectivity.\textsuperscript{1,2} Applications of optical sensing have advanced from analysis of the polarisation state of light, including machine vision, meteorology, remote sensing, biomedical imaging, and industrial control.\textsuperscript{3–13,13,14}

The process of ToF imaging is an active imaging technique providing new information within images, such as range of objects and detection of moving objects. Images are encoded by the phase difference of excited and incident signals.\textsuperscript{15} Transient images developed reveal the object as a sequence of frames, with each corresponding to the instantaneous impulse response of the scene, first implemented back in 1983 by Abramson.\textsuperscript{16} Recently, ToF imaging has had several advancements\textsuperscript{17–19} and proved to be a useful tool for solving many problems and facilitating new applications such as transient imaging.\textsuperscript{20–22} Heide et al.\textsuperscript{21} used a Photon Mixer Device(PMD) as part of an Indirect ToF system, making ToF development more affordable (approximately $1500).\textsuperscript{21} Further improvement by Lin et al.\textsuperscript{23} was developed through an alternative processing algorithm, resulting in successful reconstructed transient images.

The addition of a polarisation analyser\textsuperscript{24} with the low-cost ToF system\textsuperscript{21,23} has derived a 3D image through a scattering medium. Kien Nguyen\textsuperscript{25} designed and simulated a polarimeter using the low cost ToF imaging system.\textsuperscript{21,23} The output of such a system could produce a ToF profile of a scene with measurement of polarisation properties, simulated with General State Contrast imaging scheme to visualise the functionalities that a transient imaging polarimeter would provide.\textsuperscript{25}

The objective of this study was to further the work by Nguyen\textsuperscript{25} by developing a ToF polarimeter, with the capability of controlling the polarisation state of the illuminator and receiver capable of obtaining polarized ToF images. Benefits of being able to integrate polarimetry into a ToF system was illustrated through enhancing transient images with additional contrast and distinction optimization applied from the polarisation state of light. This will increase the utility of depth information.

II. Time-of-Flight transient imaging

A ToF system provides imaging through resolving distances between the system and different points of the scene. The system requires a round trip of light propagation, from an active modulated light source to a scene reflecting back to the image sensor.\textsuperscript{25,27} This resolved distance profile is then used to create a depth map of the scene, producing additional insights compared with conventional imaging. ToF imaging systems encompasses two types. Direct ToF measures the round trip time of flight for a short-pulsed beam to echo from an object.\textsuperscript{15} Indirect ToF works by measuring the phase delay between the incident and modulated signals to determine time of flight.\textsuperscript{15,28} Transient imaging is an application of ToF imaging systems, revealing the object scene as a sequence of frames, corresponding to the instantaneous impulse response of the object scene. This is achieved through Indirect Phase-based ToF imaging, also known as correlation-based ToF imaging. The round trip time is indirectly determined from a time-gated measurement of the incident light intensity, with a working range around ten to thirty meters.

An Indirect ToF measuring set-up is illustrated in Figure(1a). A modulated light source (a), based on Lasers or LEDs within the infra-red spectrum emits a temporally-coded illumination pattern $g(t)$. This is transmitted through (b) an optical diffuser, allowing emitted light to scatter throughout a scene. The echo back-reflected light by the scene is gathered by (c) a collection of lens’ into the sensor, represented by the solid-state ToF sensor (d). The array of photo-detectors (pixels) composed with the sensor correlates the incident signal $s(t)$ with a demodulation signal $f(t)$ measuring the time of flight required for light to travel between the illumination source, scene and back to the sensor.
Operated in amplitude modulated continuous-wave (AMCW) mode, the ToF camera detects phase shifts between the illumination and the reflection. A broad bandwidth is utilised to reduce phase ambiguity. The fast, active light source illuminates a scene with a time-varying signal or waveform. Described as sinusoids, the temporally-varying illumination signal is \( g(t) = g_1 \cos(\omega t) + g_0 \). AM frequency \( \omega \) of the modulated light emitted is in the order of 10-130 MHz with constants = \( g_0 \), \( g_1 \). An object within the scene at a distance \( d \) reflects part of the illumination back to the camera sensor. \(^{28-30}\)

Equation (1) is the signal incident on the sensor containing an ambient term \( b = \alpha g_0 + \beta \) with the constant background \( \beta \). The amplitude \( \alpha \) combines the square distance falloff related to the inverse-square law of Light \( g_1 \), as well as the albedo that is the measure of reflectance or optical brightness on a scale from zero to one of the object. The received signal phase is shifted by \( \phi = -2d\omega/c \), due to the propagation distance between the source and any particular point in the scene. \(^{28-30}\)

\[
s(t) = \alpha \cos\left(\omega\left( t - \frac{2d}{c}\right)\right) + b
= \alpha \cos(\omega t + \phi) + b
\]  

(1)

Estimation of the phase shift \( \phi \) and the scene depth image is the output of AMCW Indirect ToF Imaging. This is determined by demodulation of the temporally-varying signal incident on the sensor \( s(t) \), by a sinusoidal function \( f(\psi)(t) = \cos(\omega t + \psi) \). This demodulation is independently implemented by periodically directing photoelectrons into one of two “buckets” within each sensor pixel, capable of measuring both its phase delay and amplitude. When both the illumination and sensor modulation frequencies are matched the ToF camera operates in homodyne mode. \(^{28-30}\) The integrated sensor measurements can be modelled by accounting for a finite exposure time \( T \). This acts as a temporal low pass filter over the demodulated sensor signal and therefore, assuming the \( T \gg \frac{1}{\omega} \), the measured intensity is

\[
i(\psi(t')) = ((f(\psi)(s(t))) \ast rect(t')) \approx \frac{\alpha}{2} \cos(\psi - \phi).
\]  

(2)

The depth phase delay angle \( \phi \) and distance \( d \) is computed from sampled measurements of \( i(\psi(t')) \). Four quadrant phase values \( C_1 - C_4 \) step the relative phase difference between illumination and sensor by \( \frac{\pi}{2} \) (i.e. \( \psi = \{0, \frac{\pi}{2}, \frac{3\pi}{2}, \pi\} \)). These four different electric charge values as shown in Figure(1b), have 90 degree phase delays from each other. Using the sampled measurements the phase angle between illumination and reflection, \( \phi \), and the distance, \( d \), are computed according to the following equations.

\[
\phi_{\text{est}} = \tan^{-1}\left(\frac{i_{\frac{3\pi}{2}} - i_{\frac{\pi}{2}}}{i_{0} - i_{\pi}}\right) \quad \text{and} \quad d_{\text{est}} = \frac{c\phi_{\text{est}}}{2\omega}
\]  

(3)

These measurements are used to calculate amplitude \( \alpha_{\text{est}} \) as per the measured pixel intensity and constant background \( \beta_{\text{est}} \) offset. \(^{28-30}\)

\[
\alpha_{\text{est}} = \frac{1}{2} \sqrt{(i_{0} - i_{\pi})^2 + (i_{\frac{3\pi}{2}} - i_{\frac{\pi}{2}})^2} \quad \text{and} \quad \beta_{\text{est}} = \frac{1}{4} \left(i_{0} + i_{\pi} + i_{\frac{3\pi}{2}} + i_{\frac{\pi}{2}}\right)
\]  

(4)
III. Active Image Polarimetry

Polarimeters are scientific optical instruments utilised for sensing, measuring and characterizing the polarized state of light or materials. Through imaging polarimetry, polarisation states are mapped across a scene of interest. This builds an extensive 2D description of the polarization properties within an image.

Mueller calculus is used in most polarimeters for analysing polarization, whereby the polarization state of light is represented by the Stokes vector \( \mathbf{S} \), containing four elements as per Equation (5). Stokes vector elements match to the combination of intensity measurements \( I_i \) belonging to different polarised directions. The total intensity of light is \( s_0 \), which is the first element equal to the sum of horizontal and vertical linear polarisation. Second, third and fourth elements \( (s_1, s_2, s_3) \) are intensity differences, in respective order, between horizontal and vertical linear polarisation, between 45° and 135° linear polarisation and between left and right circular polarisation. The four elements are frequently normalized with \( s_0 \), valued from +1 to −1.

\[
\mathbf{S} = \begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \\ \end{bmatrix} = \begin{bmatrix} I_H(0) + I_V(90) \\ I_H(0) - I_V(90) \\ I_{45} - I_{135} \\ I_L - I_R \end{bmatrix}
\]

(5)

The Mueller matrix \( \mathbf{M} \) is a real \( 4 \times 4 \) matrix describing the polarisation state of light transformation characteristic within a sample. The sample can be a surface, a polarization element, an optical system or a scene with some light/matter interaction that produces a reflected, refracted, diffracted, or scattered beam of light. Transformation of the incident polarisation states due to a material is characterised by the Mueller matrix \( \mathbf{M} \) transforming the incident Stokes \( \mathbf{S}_{in} \) into the exciting Stokes vector \( \mathbf{S}'_{out} \) as per Equation (6).

\[
\mathbf{S}'_{out} = \mathbf{M} \cdot \mathbf{S}_{in} = \begin{bmatrix} m_{0,0} & m_{0,1} & m_{0,2} & m_{0,3} \\ m_{1,0} & m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,0} & m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,0} & m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} I_H(0) + I_V(90) \\ I_H(0) - I_V(90) \\ I_{45} - I_{135} \\ I_L - I_R \end{bmatrix}
\]

(6)

Incident \( \mathbf{S}_{in} \) Stokes elements are related to the four elements of \( \mathbf{S}'_{out} \) by \( \mathbf{M} \) elements. Excited polarization state of light can be determined for an arbitrary incident polarization state if the Mueller matrix of an element or scene is known. Given the Mueller matrix \( \mathbf{M} \), the Stokes vector \( \mathbf{S}'_{out} \) of the exciting polarised light can be calculated with Stokes vector \( \mathbf{S}_{in} \). For a sequence of polarization elements \( n = 1, 2, \ldots, N \), the Mueller matrix \( \mathbf{M} \) along the beam path, is calculated by the right-to-left product of the individual matrices \( \mathbf{M}_n \), according to the following Equation (7).

\[
\mathbf{M} = \mathbf{M}_N \cdot \mathbf{M}_{N-1} \cdot \ldots \mathbf{M}_n \cdot \ldots \mathbf{M}_2 \cdot \mathbf{M}_1
\]

(7)

A. Optical Polarimetry Devices

LINEAR POLARIZER A linear polarizer produces a beam of light with an electric vector vibrating primarily in a single plane, when placed in an incident unpolarized beam. This device provides control of the polarisation state of light to be generated or analysed by adjusting its transmission axis. The Mueller matrix \( \mathbf{M}_P(\theta) \) of a linear polarizer, with angle of transmission axis \( \theta \) is shown below.

\[
\mathbf{M}_P(\theta) = \frac{1}{2} \begin{bmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta - \sin^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\]

\[
\mathbf{M}_R(\delta, \theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a^2 + cb^2 & (1-c)ab & -bd \\ 0 & (1-c)ab & b^2 + ca^2 & ad \\ 0 & bd & -ad & c \end{bmatrix}
\]

(8)

LINEAR RETARDER A linear retarder changes light into two orthogonal linear polarized components, with different optical path lengths producing a shift in phase between them through the principles of birefringence. Made from birefringent, uniaxial materials with two independent refractive indices are exhibited along two different optical paths. The refractive indices are termed the ordinary index, \( n_o \), and the extraordinary
The Mueller matrix of a linear retarder is $M_R(\delta, \theta)$ with retardance $\delta$ and fast axis at $\theta$ with respect to the horizontal plane, where $a = \cos 2\theta$, $b = \sin 2\theta$, $c = \cos \delta$, $d = \sin \delta$. Typically the fast axis is set at angle ($\theta = 0^\circ \text{ or } 45^\circ$) to the horizontal axis.

### B. Mueller Matrix Polarimeters

The polarimetric scattering properties of objects and materials can be examined through Mueller matrix polarimeters (MMPs). These instruments return greater information than a passive (Stokes) polarimeter; including measurement of depolarization providing increased image information and contrast that isn’t apparent without polarization.  

![Image from Guillaume Anna](image_url)

Shown in Figure(2), light from the source passes through a polarization state generator (PSG), which made from a collection of linear retarders and polarizers is sequenced to form an elliptical diattenuator. The PSG, characterised by its Stokes vector $\mathbf{G}$, creates an arbitrary elliptical polarization state incident beam on a scene. Typically the fast axis is set at angle ($\theta = 0^\circ \text{ or } 45^\circ$) to the horizontal axis.

The incident light on the scene is transformed and excited by the scene Mueller matrix, which is then resolved within a polarization state analyser (PSA), characterised by the stokes vector $\mathbf{A}$. Measuring the intensity backscattered by the scene is received at the sensor pixels. Described through Equation(9), $I_{x,y}$ is the intensity received at the sensor pixel $(x, y)$, $I_0$ is the number of photons and $\eta$ is the conversion efficiency between photons and sensor electrons. The superscript $T$ denotes matrix transposition.

$$I_{x,y} = \frac{\eta I_0}{2} \mathbf{A}^T \mathbf{M}_{x,y} \mathbf{G}$$  \hspace{1cm} (9)

Through a range of illumination and analysis polarisation states, which form a maximum volume polyhedron inside the Poincaré sphere, Mueller matrix components can be determined. In all MMPs, the source is known (to within some calibration accuracy) and controlled to generate states of polarisation, with the detection system to sense states of polarisation. The most complete form of imaging polarimetry is the measurement of the full Mueller matrix for each pixel of the illuminated scene.

$$I_n = \mathbf{A}_n^T \mathbf{MG}_n = \mathbf{D}_n^T \mathbf{M}'$$  \hspace{1cm} (10)

With the $16 \times 1$ vectors in Equation(10) defined as;

$$\mathbf{D}_n' = vec(\mathbf{A}_n \mathbf{G}_n^T) = \mathbf{A}_n \otimes \mathbf{G}_n \quad \text{and} \quad \mathbf{M}' = vec(\mathbf{M})$$  \hspace{1cm} (11)

In Equations(11), the Kronecker (direct) product function is used as per $\otimes$. The $vec(\mathbf{M})$ generates a column vector, through reshaping the $\mathbf{M}$ matrix into a vector in a row-by-row sequence. The number $N$ of measurements are represented in an accumulated matrix.
\[
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N
\end{bmatrix} = 
\begin{bmatrix}
D_1^T M' \\
D_2^T M' \\
\vdots \\
D_N^T M'
\end{bmatrix} + \vec{n} = W M' + \vec{n},
\] (12)

In Equation (12), the full sequence of \(N\) measurements is described by the \(N \times 16\) polarimetric measurement matrix \(W\), where the \(n^{th}\) row is \(W_q\). The polarimetric measurement equation relates the intensity measurement vector \(I\) to the sample Mueller vector \(M'\), where \(W = (D_1, D_2, \ldots, D_N)^T\).

The \(\vec{n}\) is considered additive noise to the system, with variance of \(\sigma_v^2\). If \(W\) contains 16 linearly independent rows, then all 16 elements of the Mueller matrix can be reconstructed. When \(N = 16\), the inverse matrix is unique, with the Mueller matrix elements reconstructed from the polarimetric data-reduction equation \(\hat{M'} = W^{-1} \cdot I\). When \(N > 16\), Mueller matrix reconstruction is generally achieved by using the following pseudo inverse in the polarimetric data-reduction equation.

\[
\hat{M'} = W^+ I = W^+ W M' + W^+ \vec{n}
\] (13)

C. Polarimeter Implementation

Mueller matrix imaging polarimeters have been implemented in a variety of approaches: rotating retardation plates,\(^{39-41}\) rotating compensators,\(^{42}\) Pockels cells,\(^{43}\) and using photoelastic modulators.\(^{44-46}\) Generally, all 16 elements of the Mueller matrix that describe polarisation properties of the scene are captured. Liquid-crystal variable retarders (LCVRs) is a modern method of polarisation state control with several types of LCVR systems demonstrating accurate polarimetry.\(^{47-54}\) These systems measure Mueller matrix elements through illumination of a scene with four different polarization states. Reflected or transmitted light from the scene is recorded for each incident state with another four polarization states. By recording 16 images through 16 independent combinations of illumination and analysis, the Mueller matrix is reconstructed using the polarimetric data-reduction equation.

![Figure 3: Active Mueller Matrix Polarimeter with dual-variable-retarder technique](image)

The dual-variable-retarder technique (DVRT) provides all 16 polarization PSG-PSA combinations required, with fixed LCVRs.\(^{52}\) The DVRT system represented in Figure (3) consists of one horizontal linear polarizer followed by two LCVRs, oriented with angles \(\gamma\) and \(\theta\) respectively. The configurations associated with angles \(\gamma = \pm 45^\circ\) and \(\theta = \gamma \pm 45^\circ\) provide generation of any polarization state within the Poincaré sphere.\(^{52,55}\)

**Liquid Crystal Variable Retarders** The liquid crystal retarders within this polarimeter design allow for full electronic control of polarisation states. These electrically variable wave-plates consist of a liquid crystal material layer placed between two parallel glass windows. The birefringence of the liquid crystal is dependant upon the alignment of anisotropic nematic liquid crystal molecules that form uniaxial birefringent layers, as per a linear retarder. As shown in Figure (4), the molecules are aligned along parallel axes\(^1\) to the transmission axis, which is randomly distributed throughout.\(^{56}\)

Further alignment is achieved and adjusted by applying an electric field, via the ITO, which adjusts the orientation of the molecules.\(^{56,57}\) With no voltage applied, the liquid crystal molecules lie parallel to the transmission and exhibit a maximum index of refraction \(n_e\), resulting in maximum retardance. When the voltage increases, an electric field is applied and molecules align themselves within the field. Concurrently,
this also decreases $n_e$, therefore reducing effective birefringence and decreasing the retardance. Therefore, in order to control the retardance, a variable, low voltage waveform must be applied after calibration.

![LCVR Diagram](image)

Figure 4: LCVR diagram, revealing molecular alignment when no voltage is applied (a) and minimum retardance when voltage is applied (b) (Image from Meadowlark Optics)

IV. Results

A. Design of Time-of-Flight Polarimeter

The proposed ToF polarimeter system by Nguyen\textsuperscript{25} was based upon the low-cost ToF imaging system used in Heide et al.\textsuperscript{21} The aim of the design was to embed polarimetry functionality into the existing ToF system. Through analysis and assessment of the proposed design, several implications were discovered with the system, including equipment, safety issues and building requirements. Due to safety concerns and discontinuation of original PMD sensor, the original proposed system required a redesign.

Research of ToF systems for transient imaging determined a recent alternative design, developed by Shrestha et al.\textsuperscript{30} This system utilises a current sensor system by Texas Instruments within a custom control electronics design with a daughter board and external modulation. Open source documentation\textsuperscript{58} provides the list of hardware components, capture software and firmware for the micro-controller. Shrestha et al\textsuperscript{30} has proposed a reproducible, fully programmable ToF camera platform, providing custom waveforms and synchronization capabilities at a low cost, comparable to the original proposed design.\textsuperscript{25, 30}

Due to its recent development and current production equipment, it was determined to be a more feasible ToF system for integration with polarisation components to develop the ToF polarimeter system. Implementation time was greatly saved by reducing complexity due to the use of an open source design, providing a safe and highly feasible instrument with current production components. Integration of Shrestha et al\textsuperscript{30} design with a DVRT could provide measurement and reconstruction of 16 Mueller matrix elements with ToF images.

B. Revised System Implementation

After review of laboratory equipment, a limited number of LCVRs with matching operation spectra were found. Therefore, the original proposed system design of a Mueller matrix polarimeter based on DVRT was readjusted. The limit of two LCVRs with matching spectra operation for both (Visible LCVR: 450 - 700 nm) and (Infar Red 3 LCVR: 1200 - 1700 nm) allows for a revised polarimeter design, with one polarizer and one LCVR within the PSG and PSA. This design is shown in Figure(5) and represents a fixed position generator-analyser polarimeter.

![Block Diagram](image)

Figure 5: Block diagram of redesigned Mueller polarimeter, with a single fixed polarizer and LCVR within the PSG and PSA

This design is limited in its ability to reconstruct only nine of the sixteen components of the Mueller matrix. The stokes vector of light $(1, 1, 0, 0)^T$ is transmitted from the polarizer into the LCVR with the PSG.
The Stokes vector generated from the PSG, as per Equation (14), is determined from the LCVR Mueller matrix Equation (8) and the angle between horizontal transmission axis of the polarizer and LCVR fast axis fixed at 45°.

\[ S_{PSG} = \begin{bmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = I_p \begin{bmatrix} 1 \\ \cos \delta(V) \\ 0 \\ \sin \delta(V) \end{bmatrix} \tag{14} \]

The retardance as a function of applied voltage is \( \delta(V) \) and the \( s_2 \) component is zero for any voltage applied. Therefore, the ellipse of polarisation azimuth is constantly horizontal with all \( S_{PSG} \) polarisation states placed along a single meridian of the Poincaré sphere, highlighted red within Figure (6). As a result, the polarisation states are limited between horizontally polarized light (H), right circularly polarized light (R), vertically polarized light (V) and left circularly polarized light (L).

By controlling LCVRs and adjusting the retardance, the ellipticity of the generated states is changed.

Figure 6: PSG and PSA states of the system, shown within Poincaré Sphere, along a single meridian, between H, R, V and L.

\[ M_{PSA} = M_{pol}^{0} \cdot M_{-45}^{45} = \frac{1}{2} \begin{bmatrix} 1 & \cos \delta'(V') & 0 & \sin \delta'(V') \\ 1 & \cos \delta'(V') & 0 & \sin \delta'(V') \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{15} \]

The input Stokes vector \( S_{PSG} \) is transformed by the unknown Mueller matrix of a sample/scene \( M \), resulting in the output stoke vector \( S_{out}' \). The Mueller matrix \( M_{PSA} \) of the PSA is generated with the LCVR fast axis at \(-45°\) with respect to the transmission axis of polarizer. Here the external voltage \( V' \) is applied to the LCVR corresponding to retardance \( \delta' \). As shown, the input stokes vector \( S_{PSG} \) is transformed through the entire system producing \( S_{OUT} = M_{PSA} \cdot M \cdot S_{PSG} \) with the first element of \( S_{OUT} \) being the intensity of light detected by the camera.

With this system configuration, nine intensity images \( I \) are measured through the camera, each corresponding to a different independent polarisation state combination set by PSG-PSA. Through determination of the polarimetric measurement matrix \( W \), the Mueller matrix component images for a sample or scene are reconstructed with the polarimetric data-reduction equation \( \hat{M}' = W^{-1} \cdot I \), producing only nine components of the Mueller matrix.

For the DVRT, at least four linearly independent stokes vectors are chosen in order to form a maximum volume polyhedron within the Poincaré sphere to reconstruct 16 components of a sample Mueller matrix. Due to the redesigned system limited along a single meridian of the Poincaré sphere, at least three linearly independent stokes vectors are required to form an equilateral triangle to effectively reconstruct nine Mueller matrix elements. These independent stokes vectors, shown in Figure (6), were set based on PSG-PSA retardance values configured at \( \delta = 0°, 120°, 240° \).
C. Simulation of expected Mueller Matrix Polarimeter

Simulation of revised Mueller matrix polarimeter was developed in MATLAB to gain a deeper understanding of the system and Mueller matrix reconstruction from measured intensity images. The nine configurations of PSG-PSA settings were configured based on three revised polarisation states. The retardance values configured were $\delta = 0^\circ, 120^\circ, 240^\circ$ for PSG and PSA providing the nine configurations. The simulation utilises linear polarizer and variable retarder models to generate the system polarimetric measurement matrix $W$. An object matrix was developed with a series of generic patterns as shown in Figure(7a). Measured intensity images are calculated for each configuration utilising Equation(12). From the nine measured intensity images, the object Mueller matrix is reconstructed with the polarimetric data-reduction equation $\hat{M}' = W^{-1} \cdot I$, resulting in the Mueller matrix shown in Figure(7b). It can be seen from the simulation results, the redesigned system in Figure(5), will only be able to reconstruct the nine elements of the Mueller matrix. As per the limitation of the system with only a single LCVR within the PSG-PSA, the $\pm S_2$ stokes component cannot be generated or analysed. This results in non-reconstruction of the Mueller matrix third row and column as shown in Figure(7b).

![Simulation of expected Mueller matrix polarimeter](image)

Figure 7: Simulation of expected Mueller matrix polarimeter

D. Time-of-Flight System

The revised ToF camera, was configured for operation with MATLAB control and image capture. MATLAB operation of the camera allowed for instant control to capture depth and amplitude images. Internal and external modulation was tested. The external modulation set-up with the daughter board attached to the ToF camera was initially configured. Operation of external modulation was unsuccessful due to errors in image quality produced. It appeared that modulation was applied to the camera but not illuminating the scene. Internal modulation was successfully modulated and captured by MATLAB, as shown in Figures(8a & 8b). The depth image is colour scaled to represent the depth in meters, as shown by the colour bar. Image quality of both depth and amplitude images produced provided accurate object definition and contrast.

![Time-of-Flight System](image)

(a) ToF Depth Image - Colour scale represents depth in meters. (b) ToF Amplitude Image - Gray scale represents reflected IR amplitude.

Figure 8: Internal Modulation ToF Images
E. Polarimetry Calibration

In order to control the polarisation state of light using LCVRs within the Mueller matrix polarimeter, polarimetry calibration was required. Through characterization of LCVRs, the retardance for a specific wavelength of light can be selected by applying respective voltages. Utilising the theory of calibration, the voltage-retardance function can be obtained for complete control of polarisation states. The method of LCVR calibration involves using two linear polarizers with respective transmission axes perpendicular of each other, the variable retarder placed between the polarizers with its optical axis at 45° from the horizontal (Figure 9) and a final stage of “phase unwrapping” on experimental data in order to obtain the voltage-retardance function.

![Figure 9: Block diagram of LCVR calibration set-up, used to characterise a LCVR](image)

The optical system in Figure (9) affects the stoke vector of light through Mueller equation \( S_{\text{out}} = M_{\text{sys}} S_{\text{in}} \). The input stokes vector \( S_{\text{in}} \) is transmitted by the source. The output stokes vector \( S_{\text{out}} \) is detected at the camera. The Mueller matrices of sequenced polarisation components, form the Mueller matrix of the system \( M_{\text{sys}} = M_{P2}(90^\circ)M_R(\delta, 45^\circ)M_{P1}(0^\circ) \) calculated by the right-to-left product of the individual matrices.

The first term of the stokes vector \( S_{\text{out}} \) is the detected intensity \( I \), where by \( I = S_{0\text{out}} = A(1 - \cos\delta) \). The retardance of the LCVR is given by \( \delta \) with \( A \), a constant, dependent on experimental parameters including absorption and excitation ratio of the linear polarizers. The maximum intensity, \( I_{\text{max}} \), occurs when \( \cos(\delta) = -1 \), as a result of substituting, \( A = \frac{I_{\text{max}}}{2} \). This results in \( I = \frac{I_{\text{max}}}{2}(1 - \cos(\delta)) \), rearranging for Equation (16).

\[
\delta = \cos^{-1}(1 - \frac{2I}{I_{\text{max}}})
\]  

Experimental Results of LCVR calibration

The light intensity and voltage applied to the LCVRs (Figure 10a) show the average pixel intensity captured with amplitude images using the ToF camera. These measurements were recorded with the impeded light source providing IR illumination at 850 nm. By applying Equation (16) to the measured values of intensity, retardance as a function of applied voltage to a LCVR was calculated as shown in Figure (10b). This figure shows the curve of retardance variation (in degrees) with LCVR applied voltages. These curves are phase limited to values between 0 and \( \pi \) due to Equation (16), “wrapped” between 0° and 180°. The method of “phase unwrapping” is necessary to acquire correct retardance value as actual values cannot be extracted directly from the physical signal. The analysis process of “phase unwrapping” is performed on the experimental data to indirectly obtain the original continuous function of the applied-voltage-to-retardance relationship by removing discontinuities known as “phase jumps” as shown in Figure (10b).

After the “phase unwrapping” procedure, the full range of optical retardance variation with the voltage applied is shown as continuous curve in Figure (10c), which reveals the retardance for each LCVR calibrated. The characteristic plots between retardance and voltage applied for the LCVRs provides the specific voltages required for the Mueller matrix polarimetric measurements. The accurate relationship between retardance and applied voltage provides feasible polarisation state control. As shown, the retardance control is limited (0° ↔ 260°) due to the specific IR illumination at 850 nm. Optical retardance of these devices depends on the wavelength, as studies show retardance can change between individual pixels. Therefore, further calibration should be performed for each individual pixel of the ToF Camera. The compelling similarity between these calibrations in comparison to experimentation work with visible wavelengths, other work with LCVR and higher wavelength LCVR manufacture calibrations further indicates result validity. Analysis of the LCVR characterizations determined the voltage values, were feasible for the original retardance values at \( \delta = 0^\circ, 120^\circ, 240^\circ \). Another system was tested with values of \( \delta = -20^\circ, 120^\circ, 220^\circ \).
Figure 10: Experimental Results of LCVR Calibration

F. Partial Mueller Polarimeter Results

In order to test the partial Mueller polarimeter, a standard image capture system utilising a Charge Coupled Device (CCD) camera was implemented instead of the ToF Camera. The redesigned partial Mueller polarimeter with CCD camera and white light source was developed using the two electronically controlled LCVR, as per Figure(5). A MATLAB program developed, adjusted and applied appropriate voltages for PSG-PSA polarisation states, whilst the nine independent pairs of polarization state images were captured. Reconstruction of the scene/sample Mueller matrix was performed in MATLAB, providing the nine elements of the Mueller matrix.

The reliability of the polarimeter was verified through several sample measurements of optical polarisation components. These experimentally obtained Mueller matrices provide information in terms of polarimeter precision and define the accuracy of polarimetry. The comparison between experimental data and theoretical Mueller matrices obtained from the Handbook of Optics were determined.

The optical polarisation components sampled were a linear polarizer and quater-wave plate. Normalised images of the Mueller matrices were obtained for a linear polarizer with a transmission axis in both horizontal and vertical positions as shown in Figures(11a & 11b). The quater-wave plate Mueller matrix with the fast axis of the retarder at 45° was also obtained as shown in Figure(19). Equations(17 & 18 & 19), show comparison of theoretical Mueller matrices $M^T$ and experimental Mueller matrices $M^E$.

\begin{equation}
M^T_{HLP} = \begin{bmatrix}
1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\end{equation}

\begin{equation}
M^E_{HLP} = \begin{bmatrix}
1 & 0.92 & 0 & 0.16 \\
0.61 & 0.42 & 0 & 0.1 \\
0 & 0 & 0 & 0 \\
0.04 & 0.03 & 0 & 0.2 \\
\end{bmatrix}
\end{equation}

(a) Horizontal Linear Polarizer. (b) Vertical Linear Polarizer. (c) λ/4 wave plate 45° fast axis.

Figure 11: Polarisation component Mueller matrix images
\[
M_{V_{LP}}^{T} = \begin{bmatrix}
1 & -1 & 0 & 0 \\
-1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
\[
M_{V_{LP}}^{E} = \begin{bmatrix}
1 & -0.85 & 0 & -0.035 \\
-0.74 & 0.69 & 0 & 0.01 \\
0 & 0 & 0 & 0 \\
0.18 & 0.2 & 0 & 0.016 \\
\end{bmatrix}
\]

\[
M_{Q\omega_{LR}(45^\circ)}^{T} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
\end{bmatrix}
\]
\[
M_{Q\omega_{LR}(45^\circ)}^{E} = \begin{bmatrix}
1 & -0.045 & 0 & 0.039 \\
0.062 & -0.041 & 0 & -0.293 \\
0 & 0 & 0 & 0 \\
-0.147 & 0.562 & 0 & -0.013 \\
\end{bmatrix}
\]

As per simulation results, only nine elements of the sixteen the Mueller matrix were reconstructed. Comparison of these results with theoretical matrices reveal the quality of polarimetry implemented. The systematic errors are similar to other results in literature. Errors in components of the Mueller matrices as shown, may be due to the use of white light and the LCVR calibration over the broad wavelength range. These results could improve with the use of the ToF system, illuminated and calibrated at exactly a wavelength of 850 nm.

G. Time-of-Flight Partial Mueller Polarimeter Results

The redesigned ToF Mueller matrix polarimeter has been developed using the two electronically controlled LCVRs, as per Figure(5). The system has provided incorrect results without correct focusing to capture the ToF Mueller matrix images.

V. Conclusion

Throughout this project, the ToF transient imaging system was successfully implemented and its polarisation processing was understood thoroughly. The implementation of polarisation state control with the revised ToF system was successfully designed and partially implemented using a previously proposed design. The experimental design, set-up and operation of a ToF polarimeter, consisting of a pair of LCVRs, has been described. A reduced-dimensionality Mueller polarimeter that utilised the existing hardware was designed. By controlling the voltage applied to LCVRs, only nine elements of the full sixteen Mueller matrix of a scene/sample could be obtained. MATLAB software developed provided control of instruments and image processing. Through calibration of LCVRs, accurate polarisation states were produced, enabling depth and amplitude capture of nine polarisation combinations. Through the polarimetric data-reduction equation, reconstruction of nine Mueller matrix elements was successful. As a result, both depth and amplitude image Mueller matrices show a variety of enhancements but still requires further focussing to obtain a clear result. This includes additional contrast and distinction optimization applied from the polarisation state of light increasing the utility of depth information. The present system allows us to obtain depth and amplitude images with the polarization properties of a static scene or sample.

VI. Recommendations

Further development of this project should implement external modulation with the ToF System. This would investigate the refinement of images with greater detail through increased modulation. Further polarisation control could be implemented with the use of four LCVRs to obtain full reconstruction of the Mueller matrix.

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