An acoustic travelling wave excitation system (TWS) has been developed by Defence Science and Technology Group for the investigation of high-cycle fatigue issues related to gas turbine engine bladed disks (blisks). To date, a simple research blisk has been tested and used to validate the ability of the TWS to extract the modes and natural frequencies of blisks. This study outlines the verification and validation of the TWS for use on a real, in-service blisk. The natural frequencies and problematic ranges of harmonic excitation were found with an error of 3% from FEM predictions. Furthermore, the system was found to be robust towards any errors in the experimental set-up process, suggesting operation of the acoustic TWS is much simpler than other methods of blisk excitation. Promising results were also found for application of this system in evaluating the effect of mistuning and blending repairs on the change in harmonic response. Finally, potential was identified for use of the TWS in non-destructive investigation of material properties by the application of FEM model calibration.

I. Introduction

Compressor and turbine rotors are core components of the gas turbine engine, and thus considerable research and development has been directed towards their structural and aerodynamic improvement. Traditionally, compressor and turbine rotors are constructed from two main parts; the rotor disk and the rotor blades. These rotor blades can be secured to the disk in a number of ways; the rotor disk and the rotor blades. These rotor blades can be secured to the disk in a number of ways, two of which are shown in Figure 1a. The ability to add and remove blades provides an advantage for maintainability, however, this design feature has a number of disadvantages. Namely; (a) the requirement for heavy joints, (b) a number of complex parts, (c) increased drag from the discontinuities brought about by individual blade connections, and (d) stress concentration at blade connection points.¹

![Figure 1: Comparison between traditional rotor and modern blisk.](image)

In order to eliminate these disadvantages, many engine manufacturers are now opting for an entirely new approach to the design of compressor and turbine rotors. Bladed disks (blisks), also known as integrally bladed rotors (IBRs) are one-piece components that are usually machined from a single billet or friction welded together as seen in Figure 1b. Blisks provide great advantages to modern gas turbine engines as their shape can be aerodynamically and structurally optimised to provide dramatic weight savings and...
increases in efficiency. However, as a result of their lightweight design, blisks tend to exhibit poor damping characteristics, thus making these components prone to excitation from vibrations.\cite{4}

The excessive vibration of excited engine components can often lead to a dangerous phenomenon known as high cycle fatigue (HCF) (A detailed explanation of which can be found in section II.B). Excitation of a blisk at its natural frequencies will cause dramatic increases in the response amplitude. Although these deformations are small, they occur cyclically at rapid rates often in excess of 10 kHz. This type of response can quickly induce a large number of stress cycles in excess of $10^7$, potentially causing catastrophic failure of engine components without warning.

This study will aim to answer the following research questions: (1) Can the travelling wave system be used as a practical method to extract the modes and natural frequencies of gas turbine engine bladed disks? (2) To what extent can the travelling wave system be used to indicate the effects of mistuning on gas turbine bladed disks? (3) How effective is this system as a tool for the analysis of high cycle fatigue?

II. Literature Review

Although HCF has plagued the turbo-machinery industry for some time,\cite{5} the introduction of blisks and IBRs have increased the need for in-depth research into HCF and as a result extensive literature can be found on this subject and its associated areas.

II.A. Resonant Blisk Behaviour

The behaviour of cyclical systems when exposed to vibrating forces was of interest to researchers even before the invention of the gas turbine engine. Campbell\cite{6} provides a comprehensive study of vibration phenomena pertaining to steam-turbine disk wheels which can be applied to modern blisks. In 1923, the General Electric Company commissioned an investigation into fatigue failure due to disk vibration. Campbell headed this study and developed the notion of mode shapes and critical engine speeds.

By use of magnetic excitation, Campbell was able to show that at natural frequencies, disks would exhibit deflection patterns known as ‘modes’. These modes were identified by scattering sand over the disk and observing where the grains would disperse and collect as seen in Figure 2. It was found that sand would collect at the points with little or no vibration, these areas being termed ‘nodes’. Each mode can only be made from an even number of nodes as, for each area of deflection, the areas across the adjacent nodes must deflect in an opposite manner. For simplification, the ‘nodal diameter’ is defined as the number of axes about which deformations occur; or half the number of nodes. Mode shapes can be further defined by the manner in which vibrational displacement occurs. With respect to individual blades on the disk, the mode could be flapping, bending or torsional.\cite{7}

Another concept introduced by Campbell was that of engine critical speeds. A rotor will encounter upstream disturbances in the engine flow path from the stator vanes as it rotates; the number of upstream disturbances to the rotor is known as the ‘engine order’. The engine order becomes an extremely important parameter when predicting critical engine speeds. A critical engine speed occurs when a disk is excited at one of its natural frequencies, usually from upstream stator vanes. In summary, a larger number of stator vanes will increase the engine order thus decreasing the required engine speed to excite a given natural frequency.

II.B. High-Cycle Fatigue

The importance of HCF to the safe operation of modern jet engines is reflected in the extensive literature devoted to the subject. Cowles\cite{8} provides an in-depth discussion on the of HCF on the gas turbine industry. In 1996 Pratt and Whitney\cite{8} estimated that 24% of all military jet engine failures could be attributed to HCF with the primary failure location being rotor blades. In 1998 this figure increased, with 55% of all US Air Force (USAF) fighter engine critical issues being attributed to HCF.\cite{9} Cowles classifies the causes of HCF into four distinct areas. (1) Aerodynamic excitation; from engine flow path pressure perturbations. (2) Mechanical vibration; caused by rotor imbalance and rub. (3) Airfoil flutter; due to aero-mechanical blade instability. (4) Acoustic fatigue; normally associated with sheet metal components in combustors. This study will only consider aerodynamic excitation as it is the primary method of blisk excitation. However,
the travelling wave system (TWS) could be applied to other engine components to simulate different types of HCF sources.

Further literature can be found specifically relating to the matter of blisks and HCF. Pierre and Castanier\textsuperscript{5} provide a detailed survey of current bladed disk vibration modelling and its application to HCF. From this research, it has become apparent that with the reduction in damping brought about by blisks, HCF issues are increasingly prevalent. This is especially the case for military aircraft engines and even more so, rotary wing engines. Military aircraft engines are usually operated through their entire speed range significantly more frequently than their commercial counterparts. In doing so, the engine may be exposed to critical engine speeds as described in section II.A. Furthermore, aircraft that are often exposed to environments with significant risk of damage from foreign object debris (FOD), experience a heightened risk of the mistuning effects that will be described in section II.C. The vast majority of engines prone to FOD ingestion are utilised in helicopters, with HCF problems exacerbated by the drive for lighter and smaller engines for rotary wing applications\textsuperscript{10}. When FOD damage causes crack initiation and mistuning on rotating components, the situation can quickly become dangerous when combined with modal excitation producing upwards of $10^6$ stress cycles within one hour\textsuperscript{11}.

II.C. Mistuning

A perfectly ‘tuned’ blisk is one with a completely cyclically symmetrical profile with identical material properties in each cyclic sector. In reality, this is not possible to achieve due to manufacturing tolerances, defects, and most commonly, non-uniform damage. Mistuning can bring about a phenomenon known as mode localisation. This causes certain mode shapes to occur in a small, concentrated area on the blisk, producing increased vibration amplitudes which heighten the risk of HCF.\textsuperscript{5} In addition, mistuning can change the natural frequencies of the blisk, which may present an issue if the engine has been designed to omit particular critical engine speeds from its operating range.

Slater et al\textsuperscript{12} present an overview of the emerging directions of mistuning analysis as well as the various ways in which mistuning can be accounted for in system equations of motion. Hou and Cross\textsuperscript{13} present a method of including mistuning into the system by addition of a stiffness and mass mistuning term into the blisk lumped-mass equation of motion. Hou\textsuperscript{14} also provides a mistuning model with the specific inclusion of blade cracking. More complex, three-dimensional mistuning models are available that do not assume a lumped mass parameter model. However, these still require simplifying assumptions and as a result possess significant limitations\textsuperscript{15}. The derivation and direct application of these mistuned equations of motion is outside the scope of this study.

II.D. Excitation Systems

A number of physical excitation systems have been developed to conduct practical experiments on resonant blisk response. Campbell\textsuperscript{6} first used magnetic excitation whilst analysing the harmonic response of steam turbine disk wheels. This form of excitation is obviously limited to magnetic test articles, which is not widely applicable to modern gas turbine blisks; being commonly constructed from titanium and other non-ferrous materials.\textsuperscript{4} Despite this, magnetic excitation is still readily used for research purposes\textsuperscript{16}. Other systems such as oil-jet spin rigs\textsuperscript{17} and hammer excitation\textsuperscript{18,19} are also in use but require expensive and often complex mechanical equipment prone to design issues. Oil jet excitation is achieved by rotating a blisk up to engine speed, and projecting a constant stream of oil onto a concentrated area of the blades to emulate the aerodynamic excitation produced by an upstream stator vane. Hammer excitation is simply striking the object with a small probe at a high frequency in order to provide excitation via physical contact.

Acoustic excitation is a relatively new method of analysis which provides a force on the blisk through pressure (sound) waves. By emitting the pressure wave at a specific frequency, the blisk will be excited at that same frequency. The dynamic response can then be analysed through use of a laser doppler vibrometer. The natural frequencies can be observed through spikes in the amplitude response of the blisk. The engine order excitation can be replicated through a control system that provides a phase offset for each individual speaker connected to each blade.

Jones and Cross\textsuperscript{20} investigated the use of an acoustic travelling wave system as an alternative to magnetic excitation with promising results. The major issue proposed was the lack of forcing amplitude on the blades to obtain a clear dynamic response. A major limitation of the travelling wave system is the negation of
centrifugal stiffening. In reality, a blisk would be rotating at significant speed, causing a ‘stiffening’ effect, raising its natural frequencies. Accounting for this is currently an active area of research.

### III. Previous Work

A significant amount of work has been conducted in preparing the TWS for testing on a real blisk. This included designing the system, testing on a research blisk and implementation of a real blisk to the system. This work is detailed in the following section prior to the presentation and analysis of results.

#### III.A. Acoustic Travelling Wave System

The TWS consists of three main sub-systems; excitation, blisk mounting and data acquisition. The blisk is mounted on a compressed air ‘floating vibration free table’ and is excited by speakers directed at each blade via identical-length PVC tubing. The speakers are powered by a set of amplifiers capable of producing sound waves in excess of 90 dB. The output of the amplifiers is controlled by a function generator which provides a phase offset for each speaker. This phase offset produces a travelling wave excitation similar to that used by Jones and Cross. A Polytec laser doppler vibrometer (LDV) measures response amplitude of the test article through measuring out-of-plane velocity at various scan points set by the user. The data acquisition hardware connected to the LDV scanning head is also provided a reference signal from the function generator to improve the quality of results.

The modes of the research blisk were extracted using the TWS and compared to FEM results in previous work conducted by DST Group. Good correlation was found with predicted FE and experimental results with a maximum error in the natural frequencies of 3.57%, hence further validation for use on a real engine blisk has been sought.

#### III.B. Research Blisk

The research blisk used in the development of the system was made from stainless steel using an electronic discharge machining (EDM) process. The blisk was manufactured with a uniform thickness of 6 mm and 12 blades. Circular holes at the blade roots were also included in the design to reduce stress concentration areas and prevent inter-blade coupling which could lead to unexpected results.

#### III.C. Real Blisk Implementation

In order to test a real blisk using the TWS, a mounting sub-system was designed and manufactured by the author as seen in Figure 3b. An assembly drawing of this design can be seen in Appendix A. It was a requirement of the design to be compatible with the existing mounting chuck, speaker system and scanning laser vibrometer. A mounting plate for the nozzles was machined from a billet of 7075 T6 Aluminium using a computer numerical control (CNC) mill. The dimensions of this plate were calculated using a three-dimensional CAD model of the test article to obtain a nozzle angle normal to an identical surface on each blade. The selected area was required to be sufficient for even distribution of a force, yet provide a substantial enough moment arm for excitation by relatively weak pressure waves.

Overall, the rig has been designed to ensure uniformity throughout with the least potential for measurement error during setup. Designs which allowed for greater freedom of nozzle position were considered, however uniformity of the experimental parameters was assessed as a priority; despite this, limited scope still exists within the system to investigate the effect of forcing on different locations of the blades.

![Figure 3: Comparison between original research blisk set up and real blisk assembly.](image-url)
IV. TWS Experimental Results

Experimental testing using the TWS was conducted in two phases. The initial phase was used to quantify the various sources of error in the system and provide preliminary results for use in more detailed FEM. The second phase of testing was used to obtain results with increased accuracy and precision whilst examining the effect of different engine orders. Instructions for operation of the TWS can be found in Appendix B.

IV.A. Design of Experiments (DOE) Approach

A DOE was implemented, whereby the parameters affecting the results (average change in natural frequency and amplitude) were ranked by a Plackett-Burman screening test. This methodology was chosen by balancing the relatively short period available for testing with the essential requirement for uncertainty quantification of the experiment. The various parameters of interest and their respective high and low limits are detailed in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lo Limit</th>
<th>Hi Limit</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweep time</td>
<td>30</td>
<td>40</td>
<td>s</td>
</tr>
<tr>
<td>Nozzle to blade distance</td>
<td>0.005</td>
<td>0.035</td>
<td>in</td>
</tr>
<tr>
<td>Nozzle to blade distance uniformity</td>
<td>uniform</td>
<td>non-uniform</td>
<td>-</td>
</tr>
<tr>
<td>Nozzle diameter</td>
<td>1</td>
<td>4</td>
<td>mm</td>
</tr>
<tr>
<td>Nozzle location</td>
<td>design case</td>
<td>right azimuth limit</td>
<td>-</td>
</tr>
<tr>
<td>Clamping Torque</td>
<td>16.9</td>
<td>30.5</td>
<td>N·m</td>
</tr>
</tbody>
</table>

The testing followed a 12-run orthogonal test matrix as outlined by Plackett and Burman. This matrix details the experimental settings for each test number. The results of this are discussed in Section IV.C.

IV.B. Initial Results

The frequencies and amplitudes of all peak responses found within the frequency sweep range for each run are seen in the Fast Fourier Transform plot (FFT) in Figure 4a. Overall, 14 peak responses were found for engine order one. The expected result from FEM predictions was 10 peaks as calculated by a tuned modal analysis. The results of the experiment differ from the FEM predictions by 4%. Whilst this error would appear small, the FEM results suggest spacing between the natural frequencies as little as 0.3 Hz. Considering this, the error could be viewed as significant. The difference in results could possibly be attributed to mistuning, especially considering that more than 10 peaks are observed for the TWS response, indicating some modes have experienced frequency veering. In order to quantify the effect of mistuning with respect to the relative position of the experimental and numerical frequencies, mistuning damage was introduced into the FEM, as discussed in Section VII.A.

The experimental results in Figure 4a exhibit peaks that are poorly defined, meaning that more peak responses could be hidden inside the noise of the larger peaks shown on the plot. This suggests that more experiments will need to be conducted with finer LDV measurement resolution. The testing conducted in this initial phase was at a resolution of 0.3 Hz, suggesting modes with spacing less than this value would not be observed.

IV.C. Uncertainty Analysis

A plot of the natural frequencies and amplitudes extracted using the TWS can be seen in Figures 4b and 4c. Each natural frequency and amplitude can be seen with its associated measurement error due to the sources discussed in Section IV.A. These uncertainty ranges were calculated using the Plackett-Burman screening test.

The experimental set-up error bars in Figure 4b suggest a relatively small uncertainty produced by the experimental design which is far outweighed by the measurement uncertainty contributed by the hardware. The hardware measurement error of ±1% is a characteristic of the system and cannot be changed. The experimental set-up uncertainty sources are discussed below however; given that this uncertainty range
is significantly less than the measurement uncertainty, it can be deemed acceptable. Unlike the frequency results in Figure 4b, the uncertainty of amplitude measurement shown in Figure 4c for each mode far exceeds the 1% system measurement error. The sources of this setup uncertainty are also explained below.

Figure 5 shows the extent to which each parameter considered in the screening test contributes to the errors seen in Figures 4b and 4c. The standardised effect is calculated from the T score for a 95% confidence level hypothesis test on an assumed t distribution with 6 DOF (one for each parameter). The red line is calculated using Lenth’s Pseudo-Standard Error\(^2\) where any effect greater than this can be assumed statistically significant.

It is clear from Figure 5a, that the nozzle size considerably affects the amplitude response of the blisk. Ultimately this affects the clarity of the results, as a larger amplitude response will provide more defined peaks for easier mode identification. The Pareto Plot in Figure 5b shows that the greatest effect on natural frequency location is the clamping torque. This prompted a more rigorous analysis into the boundary conditions (clamping) of the blisk, which can be found in Section V.B.

A relatively small increase of 0.5 Hz in the average natural frequency was observed as the whole assembly is tightened. This implies that some difference between the FEM and experimental results could be attributed to the boundary conditions (fixed constraints). Although the effect appears relatively small, a 0.5 Hz increase may be significant when considering the spacing between modes is as little as 0.3 Hz.

The natural frequencies of the system showed similar results where a clamping force appears to linearly increase the natural frequencies at a rate of $1 \times 10^{-4}$ Hz/N over the torque range tested. A study conducted by Şakar\(^2\) showed that a compressive axial force on a cantilever beam decreases its natural frequencies; however, a compressive force, as used in the particular setup for this experiment, linearly increases the natural frequencies. It is likely that this is due to the nature of the structure which has proven not to be harmonically comparable with a cantilevered beam.

Although only two nozzle sizes were tested, the large (4 mm) nozzle produced a nearly three-fold increase in the amplitude response of the blisk, resulting in a much clearer signal and easier identification of the peaks on the FFT. The results of the screening test provided an indication of the sensitivity of the results due to nozzle size, however more investigation is required into the large degree of uncertainty in the amplitudes as seen in Figure 4c.
A potentially significant source of uncertainty which has not been accounted for is the laser measurement resolution. This differs from the 1% standard error as the LDV FFT resolution can be set by the user. During this initial testing, all runs were conducted at a resolution of 0.3 Hz, which upon examining the close frequency spacing of higher modes can be regarded as a significant source of uncertainty. Not only is this measurement resolution important for accurate mapping of the frequency domain, it may contribute to the large amplitude uncertainty. It should also be noted that the temperature was monitored during each test and was found to have no notable effect on the results.

V. Finite Element Modelling

A significant amount of FEM was carried out for comparison with experimental results. Furthermore, FEM results were utilised to design the experimental setup. As the travelling wave system is not capable of rapidly scanning large frequency ranges, the FEM results provided an estimate of the range of interest, which for the real blisk was between 1,000 to 1,200 Hz. FEM results also provided the mode shapes expected at given natural frequencies, which aided in the comparison and correlation of results when using the travelling wave system laser holography.

V.A. Material Properties

It is expected that density and elastic modulus are non-uniform throughout the structure; however, an accurate distribution of material properties throughout the blisk is extremely complex to model and was not considered in this study. According to the OEM, this blisk is machined from a single piece of Titanium Ti-6Al-4V with a density of 4500 kg · m$^{-3}$. However, the weight of the blisk was measured as $1.9500 \pm 0.0005$ kg and the volume obtained from the laser scanned CAD model as $45.14 \times 10^{-5}$ m$^3$. The uncertainty of the volume measurement was deemed to be insignificant. This weight and volume resulted in a density calculation of $4319.62 \pm 1.11$ kg · m$^{-3}$. An elastic modulus range of 105 to 115 GPa at 20$^\circ$C was provided by the blisk OEM. A Poisson’s Ratio of 0.36 was also provided. As there was some doubt in the accuracy of these values, an in depth uncertainty analysis was conducted which can be found in Section V.D.

V.B. Boundary Condition Analysis

Initial testing on the TWS indicated a significant amount of discrepancy between FEM and experimental results could be attributed to the manner in which the boundary conditions were modelled. As a result, the effect of altering and making these boundary conditions more realistic was investigated. The experimental setup consists of a threaded steel shaft with a threaded steel nut used to clamp the blisk into place as seen in Figure 3b. This clamping force is transferred through the curvic coupling adapters so that the static loading is similar to the configuration inside of an engine.

The force applied in the boundary conditions was calculated by utilising the elastic torque-tension relationship given by Shoberg$^{25}$ as: $T = KDF$. Where T is the torque of the faster in N · m, K is a nut friction factor given as 0.2 for steel,$^{26}$ D is the thread major diameter in m and F is the ‘clamping’ force in N.

V.C. Grid Convergence Study

A grid convergence study was conducted to find the grid convergence index (GCI) and relative grid error for the mesh utilised in this study. Four meshes were used with identical boundary conditions and material properties in order to quantify the uncertainty of the modal analysis due to the mesh. The GCI was calculated for each of the first 19 natural frequencies using the process outlined by Roache$^{27}$ and adapted for use in FEM by Kwasiiewski.$^{28}$ The maximum GCI for the chosen mesh was 0.0077%. This implies a maximum grid error of $\pm 0.083$ Hz. For an engine order of 18, this error represents an engine speed range of 2.76 RPM. Modern gas turbine engines idle in excess of 20,000 RPM,$^{29}$ suggesting a 2.76 RPM error is relatively insignificant.

V.D. Sobol’ Sensitivity Analysis

In order to quantify the effect of uncertainty in the FEM material properties and boundary conditions a three-parameter Sobol’ sensitivity study$^{30}$ was conducted followed by a Monte Carlo simulation,$^{31}$ providing
a detailed representation of the sensitivity and distribution of the results due to the uncertainty of the FEM input parameters.

The material properties and boundary conditions discussed in Sections V.A and V.B provide a range of possible values the blisk may truly exhibit. As such, a sensitivity study on the effect of these properties on the natural frequencies of the system was investigated and a response map created for all possible combinations of values within the defined uncertainty ranges.

![Figure 6: Sobol testing sequence with associated Monte Carlo simulation for first natural frequency.](image)

A quasi-random sampling sequence was generated by pre-existing Sobol’ Design of Experiments code. The uncertainty ranges in Sections V.A and V.B were assumed to be ±2 standard deviations centred about the mean value and normally distributed with exception of the density range. In this case, the minimum measured value of 4318.51 kg/m³ was set as −2σ and the highest suggested value from OEM data of 4500.00 kg/m³ was defined as +2σ. The limits of the ranges were taken to be two standard deviations as it can be assumed that there is 95% confidence the true properties lie within these values.

The FEM design of experiments was created using a 20-point Sobol’ sequencing test which can be seen in Figure 6a with associated frequency results. From these points, a radial basis function (RBF) was generated using third-party code in order to create a response map.

10,000 pseudo-random test points normally distributed about the means of density, Young’s Modulus and clamping force were used to create a Monte Carlo simulation on the previously generated RBF. The results of the Monte Carlo simulation for the first mode can be seen in Figure 6b.

From this Monte Carlo simulation, response maps can be generated in order to calculate the effect of various physical parameters on the response of the system, ultimately giving a detailed uncertainty quantification of the results. This result can be repeated for each of the first 19 modes calculated in the FEM modal analysis; however, only the first mode has been presented in this case.

To summarise the result of the Monte Carlo simulation in Figure 6b, a histogram was generated which displays the probability density of the first mode natural frequency as seen in Figure 6c. This process was repeated for all 19 modes calculated from FEM, producing the associated histogram for each. The results of these repeated simulations are shown in Figure 7 where the histogram for each mode has been plotted as a probability density function.

V.E. FEM Comparison

A comparison of results between the TWS and FEM can be seen in Figure 7. It can be observed that the phase 2 results using a higher resolution scan improved the accuracy as expected. Each mode extracted
from the TWS was identified by both its natural frequency using the FFT, and mode shape using LDV Holography. It can be further observed that 4 modes were not obtained from the TWS. This implies that some modes may not have split due to mistuning, or the peak response was too low to be reliably obtained through the laser measurement. A potential method for confirming this would be increasing the acoustic forcing on the system, resulting in high response amplitudes and more defined peaks.

The slight amplitude rise near 1,110 Hz has been attributed to system noise due to its lack of distinct shape and low amplitude. A limited number of other mode shapes were unable to be distinguished as their amplitudes were too low and shapes too complex to create an obvious pattern. As a result not all modes extracted from the TWS were able to be assigned to their respective FEM counterparts. However, a distinct range or ‘problem region’ was successfully identified and characterised to within 4% of FEM predictions. This translates to an approximately 130 RPM engine range which is relatively insignificant for engines operating between 20,000 - 30,000 RPM. Better correlation of these mode shapes could be obtained through increasing the forcing on the blisk, making shapes more obvious and improving signal to noise ratio.

VI. Model Calibration

As the exact material properties and boundary conditions of the FEM simulation were unknown, a method known as model calibration\(^\text{43}\) was used to find the most likely values for these parameters. This method assumes obtained experimental results are correct, and provides a set of boundary condition values which produce a FEM output as close as possible to the experimental results. If the natural frequencies obtained from the TWS are assumed to be correct, these values can be used to find the most likely material properties and boundary conditions of the FE model for use in further applications such as harmonic analysis.

Firstly, a prior function is created from the previously conducted Sobol’ study, which assumes the probability of the FE model boundary conditions being a particular value is normally distributed. This can be visualised in Figure 8a, which exhibits a three-dimensional, normal probability density function (PDF).

Following this, a likelihood function is created using the first experimental natural frequency and the FEM Monte Carlo simulation. This is generated by calculating an isosurface for the experimental value on the Monte Carlo simulation. A normally distributed three-dimensional PDF is then generated around this isosurface, the resulting function being the likelihood as seen in Figure 8b.

The posterior is the result of multiplying the likelihood and prior to give an indication of the most likely values of the boundary conditions which provide a result closest to experimental observations. The posterior function can be visualised for this case in Figure 8c. After this process is iterated for all 15 experimental results (natural frequencies) found from the TWS, the posterior converges on a localised value for clamping force, elastic modulus and density of 5000±640 N, 106.97±0.93 GPa and 4360±67 kg/m\(^3\) respectively. By utilisation and refinement of this process, the TWS may be useful for NDI of material properties. It was observed that despite the OEM suggesting a density of 4500 kg/m\(^3\), the model calibration yielded a result closer to that of the density measured through weight and volume (4320 kg/m\(^3\)).

![Figure 8: Visual representation of model calibration process.](image)

VII. Mistuning and Blending Analysis

As the blisk used in this study is heavily mistuned, it was concluded that an investigation into the effect of damage on the FEM results would prove valuable. In the previous chapters the experimental TWS results
were compared to a perfectly tuned FEM model, providing an unquantified source of uncertainty.

VII.A. Mistuned Finite Element Modelling

The blisk damage was measured using vernier callipers with an accuracy $\pm 0.02$ mm. Each damage site was modelled as a simplified half-ellipse with coordinates as seen in Figure 9. Incorporation of FOD damage into the FEM model can be seen in Figure 9d. This was modelled in ANSYS workbench using a mesh similar to the tuned case. As the geometry and mesh was altered, it was decided to conduct another grid convergence study. The GCI process previously used was repeated for the damaged geometry and mesh with acceptable results.

VII.B. Blending

Blending is used to repair gas turbine engine rotor blades by filing out existing damage into a smoother, less abrupt change in blade geometry. This process rounds out notches and smooths burrs ultimately reducing stress concentrators which may have otherwise lead to crack initiation and propagation.

The most significant damage site was blended in accordance with OEM instructions. This required the blend length to be at least 4 times the depth of the impact. The damaged location seen in Figure 9b was filed out to a resulting blend seen in Figure 9c. The blend was incorporated into the FE model using an identical method as seen in Section VII.A; a larger elliptical cut out was created with rounded edges. The results of the FE simulation conducted with both the mistuned and blended geometry indicated a shift in the natural frequencies of less than 0.5 Hz and no observable difference in amplitudes. This was consistent with the results extracted from the TWS after blending, suggesting the repair was successful in not drastically altering the harmonic response of the blisk.

VIII. Conclusions

This study has concluded that the Travelling Wave System is able to accurately identify the natural frequencies of an in-service gas turbine engine blisk with a maximum frequency error of 4%. Although not all individual mode shapes were able to be correlated to their respective natural frequencies, this still provides an effective tool for evaluating problem ranges of harmonic excitation.

The results for natural frequency were found to be very robust towards errors in experimental setup; giving further weight to the ease of use of the system. The greatest effect on experimental uncertainty of the frequency was found to be the assembly clamping torque whereas the nozzle size produced the greatest effect on amplitude results.

Potential for use of this system in non-destructive investigation of material properties was identified through the use of model calibration. This process not only provides greater accuracy of numerical predictions but may provide a useful tool for parametric studies of material property effects on blisk harmonic response.

Promising results were obtained when characterising the effect of mistuning and blending repairs on blisk harmonic response. The particular repair conducted in this study verified the manufacturer’s blending limits and repair instructions from a harmonic response perspective.
VIII.A.  Limitations

As the mode shapes for this particular blisk were found to be relatively close together when compared to the previously tested research blisk, the TWS was potentially unable to identify all high end modes of the first 19 natural frequencies. However, it may be the case that due to the frequency veering of the blisk, not all 19 modes are observable in any case as only a subset of these 19 have split. As the tuned system would exhibit 10 natural frequencies, it is clear that the TWS has identified a significant amount of mistuning in the system.

In line with the previous limitation, it was found that complex mode shapes with high nodal diameters were difficult to observe from the laser holography images. This places further limitations on accurate natural frequency characterisation as not all peak responses were able to be correlated with mode shapes.

Finally, the peak amplitudes of the harmonic response measured by the TWS still possess a significant amount of uncertainty. It is predicted that increasing the blade forcing will reduce some of this uncertainty as less noise will be present in the response, providing a more reliable FFT output.

VIII.B.  Future Work

The greatest potential increase in the quality of results is likely to derive from higher acoustic blade forcing. This could be conducted by an optimisation of the TWS nozzles through a CFD analysis. During this study it was concluded that the most dramatic effect on response amplitude was caused by increasing the nozzle diameter. For this reason it may prove advantageous to investigate the underlying physical phenomena occurring at the nozzle exit and blade surface.

A more detailed FEM harmonic analysis may also prove beneficial for improved comparison and correlation of experimental and numerical results. This would include a higher fidelity FE model of the mistuned and blended blisk geometry.

In order to gain increased confidence in TWS mistuning characterisation, it would be beneficial to introduce deliberate mistuning into the blisk and re-test on the TWS. These results could again be compared to FE predictions to identify if the effects of additional mistuning are able to be characterised.

In this study, the blisk material properties were assumed to be constant throughout the structure. This may not be an accurate assumption and as such the FE predictions are likely to have a significant amount of error due to a lack of material property variation. This variation throughout the structure would add an additional, significant source of mistuning, and as such it would potentially prove valuable to investigate the effects of mistuning due to material property non-uniformities in the FE model.

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Appendices

A - Real Blisk TWS Rig Design Drawings
B - TWS Operating Instructions

References


6 Campbell, W., “Protection of Steam Turbine Disk Wheels from Axial Vibration,” *American Society of Mechanical Engineers Spring Meeting*, 1924.


