Random mistuning refers to changes in an object’s natural frequencies due to small variations in its design, such as through foreign object damage, manufacturing defects, or discrepancies in material properties. Intentional mistuning can be introduced to further change the natural frequencies of the object and reduce the effect of random mistuning. This study will focus on the intentional mistuning of bladed disks, or blisks. Blisks are rotor assemblies consisting of the blades and disk manufactured as a single piece. Ideally, all blades vibrate at the same frequency for a given engine order. However random mistuning may cause some blades to resonate whilst others do not. This difference in frequencies can cause large stress concentrations, particularly due to the unavoidable blade-to-blade coupling in blisks. In a conventional rotor assembly, single blades can be easily replaced; this is not possible for blisks. The replacement of a blisk is time-consuming and expensive, thus it is important to reduce the effects of random mistuning and prevent premature failure from occurring. Blisks can be intentionally mistuned by having a specific amount of material removed from their leading, or trailing edge tip. This may be sufficient to reduce the effects of random mistuning. An initial testing phase is conducted using a finite element model of a simple cantilevered beam representing an individual blisk blade. This is to ascertain if geometric modification is a feasible method of intentional mistuning. Notches of varying sizes are introduced into the model, and the resulting changes in frequency are observed. Results from this testing phase show that frequency increases with increasing notch size, suggesting potential for this method of intentional mistuning to be introduced in blisk design. An experimental testing phase is used to validate the computational results and also quantify the effect of random mistuning on cantilever beams. A primary testing phase uses a finite element model to determine the effect of geometric modification on the natural frequency of a research blisk. When compared to the experimental results it can be shown that this method of intentional mistuning is sufficient to offset the effect of random mistuning in these rudimentary cases.
In an ideal case, all blisk blades vibrate at the same natural frequency. However, random mistuning may cause some blades to vibrate at different natural frequencies to others. Due to the strong blade-to-blade coupling in blisks, this frequency discrepancy can increase localised increases in amplitude and stress concentrations in the blisk, resulting in failure. Since a single blisk blade cannot be replaced if damaged, it may be possible to offset the frequency change by modifying other blades on the blisk. These modifications also result in frequency changes that may be able to reduce the stress concentrations caused by random mistuning. Modifying the blades in this way is referred to as intentional mistuning. Methods of intentional mistuning will be discussed in Section II.

II. Overview of Previous Studies

Random mistuning is a subtle phenomena that can be difficult to detect and can be even more difficult to eliminate. A common solution is to reduce the effects of random mistuning by introducing intentional mistuning as aforementioned. Castanier and Pierre (2002) identified through use of a lumped parameter model that intentional mistuning could make a rotor less sensitive to random mistuning. In this case, intentional mistuning was introduced by altering blade stiffnesses in rotationally periodic patterns. These patterns exploit the cyclic nature of the blisk and are produced using Eq. (1)

\[ \Delta_n = A \cos \left( \frac{2\pi h(n-1)}{N} \right), \quad 1 \leq h \leq \text{int}[N/2] \]  

where \( \Delta_n \) is the intentional mistuning value for blade \( n \), \( A \) is the amplitude, \( h \) is the integer harmonic of intentional mistuning, and \( N \) is the total number of blades. This equation is used with Eq. (2) to determine the nominal stiffness of each blade

\[ k_n^i = k_0(1 + \Delta_n + \delta_n^i), \quad n = 1, ..., N \]  

where \( k_n^i \) is the stiffness of blade \( n \) on a randomly mistuned rotor \( i \), \( k_0 \) is the nominal stiffness of the blades, and \( \delta_n^i \) is a random mistuning factor. In this way, intentional mistuning is introduced in the form of blade stiffness varying in harmonic patterns.

In particular, two configurations resulted in the highest reduction in forced response amplitude relative to the original tuned design, and these are shown in Fig. 1. The s2 configuration is simply a square-wave approximation of the h2 configuration; this had an even larger reduction in amplitude than the h2 configuration (Castanier and Pierre, 2002). For this reason, the s2 configuration will be used in the current study to assess the effectiveness of geometric modification in reducing random mistuning effects. Additionally, data obtained using this configuration will also provide a more meaningful foundation for future study in this area, particularly when considering forced response.

\[ \text{Figure 1. Intentional mistuning patterns; dark shades correspond to high stiffness (Castanier and Pierre, 2002).} \]

Kelly et al., (2011) proposed removing material from the pressure sides of the blades, as shown in Fig. 2. Dimension T1 represents the original thickness of the blade from the mean camber line, and T2 represents the same dimension after modification has taken place. It was stated that removing material from the corners of the blades is not a feasible method of intentional mistuning due to aerodynamic and manufacturing considerations (Kelly et al., 2011). However no evidence is provided to suggest why this may be the case.

\[ \text{Figure 2. Blade with material removed from pressure side (Kelly et al., 2011).} \]
Castanier and Pierre’s (2002) method does not dictate how the stiffnesses of the blades are altered from a physical standpoint; this leaves considerable flexibility for further testing of their method. The material, thickness and length of the blades are all parameters which change the blade stiffness and could be tested in conjunction with this method. Kelly et al.’s (2011) method is far more specific but operates on the principle that natural frequency can be changed via geometric modification. Both methods have influenced this study, as will be discussed below.

III. Testing and Outcomes

A. Research Question and Project Scope

The aim of this study is to investigate an alternate method of intentional mistuning to those presented above. There are presently no studies on intentional mistuning by removing material from the tips of the blades. The current study has been conducted to shed light on this idea, and determine whether it produces a sufficiently significant effect to warrant further research. The proposed method involves trimming the corners of blisk blades and determining the resulting change in natural frequency. The results obtained will indicate whether or not this is an effective method of intentional mistuning; that is, is the effect significant enough to reduce the effect of random mistuning?

To simplify the study, a model of a 12-bladed research blisk has been used, as depicted in Fig. 3. The research blisk is a sufficient approximation of an actual blisk in terms of identifying general trends in modal response from a purely structural perspective. Additionally, aerodynamic and rotational effects of the notches have been neglected. It is expected that this method of geometric modification will have a significant effect on the aerodynamics of the blisk, particularly when considering modifying the leading edge versus the trailing edge of the blades. However, the study has been simplified to only include structural effects at this point to identify if the desired effect can be created as simply and computationally efficiently as possible. Additionally, comprehensive testing at the Defence Science and Technology Group using a similar research blisk on a traveling wave rig. The research blisk was found to sufficiently represent blisk structure to conduct their work.

In order to answer the research question, the study was broken down into three testing stages. These are described in in Fig. 4.

B. Initial Testing Stage

In this stage, a modal analysis was performed on a series of simple cantilevers using FEM. The solver solves Eq. (3), the dynamic equation (no external force and neglecting damping),

\[ mx'' + kx = 0 \]  

(3)

to obtain the natural frequency. The shape of each mode is represented in a colour map that shows normalised deformation (see Fig. 5); red denoting high deformation and blue low deformation. Deformation magnitudes cannot be obtained without performing a forced response analysis.

Each cantilever represented an individual blisk blade with a different geometry. This was to determine the effect of notch size on the natural
frequencies of a given cantilever. The geometry was selected to accommodate the frequency range of the laser profilometer to be used in the experimental testing stage. The cantilever dimensions are shown in Fig. 6, and Table 1 lists the notch sizes used and the naming convention. For simplicity, \( \Delta x \) and \( \Delta y \) were equal to each other for each cantilever, so the notch was taken at a 45 degree angle. Notches were taken at 5 mm increments; this is an exaggeration of what would be expected in a real-life modification. However, this provided clear indication of the frequency trends produced by the notches.

![Cantilever geometry with parameters \( \Delta x \) and \( \Delta y \) (left) and FEM clamping geometry (right).](image)

**Table 1. Cantilever Nomenclature**

<table>
<thead>
<tr>
<th>( \Delta x ), mm</th>
<th>( \Delta y ), mm</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>Baseline</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>C5</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>C10</td>
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<td>C15</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>C20</td>
</tr>
</tbody>
</table>

A simple mesh independence study was conducted on the C10 cantilever to select an appropriate mesh size for further tests, where mesh element size was changed for each modal analysis. As seen in Fig. 7, there was a sudden increase in frequency between 1.5 mm and 2 mm mesh element sizes. The model was unable to be solved for element sizes less than 0.75 mm, and the computation time for 0.75 mm was significantly larger than for 1 mm. A mesh element size of 1 mm was selected to achieve the most accurate solution with the least computational cost.

The change in natural frequencies between each cantilever and the baseline case quantified the effect of notch size on frequency. As depicted in Fig. 8, notch size was proportional to frequency change for a given mode. This is to be expected as seen in Eq. (4)

\[
f_0 = \frac{1}{2\pi} \sqrt{\frac{E I_0}{\rho I_0}}
\]
where $f_0$ is the natural frequency, $E$ is the elastic modulus of the material, $\rho$ is the density of the material, $t_0$ is the thickness of the cantilever, and $l_0$ is the length of the cantilever. Notch size represents a reduction in length $l_0$ which increases natural frequency (Ansari and Cho, 2009).

The results of the FEM analysis are consistent with this theory; for the majority of modes, the change in frequency increased with increasing notch size. These results can be seen in Fig. 8. A quadratic trend is seen in these results; this is reflective of the relationship between frequency and length in Eq. (4).

Note that for clarity only the results of mode one are plotted; higher modes exhibited similar trends as the first mode. These cantilevers, along with the baseline case, will be tested experimentally to determine if similar trends also exist under realistic conditions.

C. Experimental Testing Stage

1. Validation of FEM method

The aim of this stage was to validate the FEM results obtained in the initial testing stage. The baseline case, as well as C5, C15 and C20 were tested. This would give a good indication of the frequency trend whilst also minimising the amount of tests to be conducted.

The initial method involved testing samples that were of the same dimensions as a single research blisk blade, with a thickness of 6 mm. However, due to the limitations of the laser profilometer, the geometry had to be modified. Frequencies obtained in FEM were of the order of tens of kHz. The laser profilometer was only able to accurately measure frequencies up to 4 kHz. As such, the geometry needed to be modified to reduce the natural frequencies of the cantilevers. First, the material was changed from stainless steel (the material of the research blisk) to aluminium. Since aluminium has a lower Young’s Modulus than stainless steel it has a lower stiffness, and therefore has lower natural frequencies. Additionally, the length of the cantilevers was increased and this further reduced their stiffness.

Each cantilever was secured in a clamping block, which was then attached to a workbench with the laser profilometer positioned above it, as seen in Fig. 9. The cantilevers were excited and the laser profilometer measured the oscillations in deformation. A Fourier transformation method was used to obtain the plot shown in Fig. 10 from deformation and time data. This amplitude spectrum shows the relative strength of each excited frequency, represented by the clear peaks.
in the data. The first and largest peak corresponds to the mode one natural frequency; subsequent peaks are the natural frequencies of higher modes.

From this data, the change in natural frequency from the baseline case could be determined. The results of the experiments are presented in Fig. 11, along with the results obtained in the initial testing stage for comparison. A similar trend as to that seen in the experimental testing stage is also seen in the initial testing stage. This suggests that FEM is a sufficient tool to model the modal response of the cantilevers as it represents realistic trends to a reasonable degree. However, the changes in frequency obtained experimentally are larger than those obtained computationally, and this discrepancy increases with increasing notch size. This may be due to sources of uncertainty in the experiment which are discussed in Section IV.

2. Random Mistuning

Three different cantilevers were tested for each notch size to determine the relative differences in natural frequencies between them. If these variations are greater than the uncertainty in the data obtained, then they are classified as random mistuning. Slight variations in dimensions and surface condition (such as scratches and impact damage) are obvious when comparing the test cantilevers. The frequency for each test cantilever is given in Fig. 12. Note that the results for cantilever C20 are omitted as limitations of the laser profilometer only permitted data collection for one single test. These results show clearly that the changes in frequency for each cantilever for a given notch size are greater than the uncertainty in the measurement. It is reasonable to assume then, that these changes are a quantitative representation of random mistuning. However, it would be prudent to conduct a more thorough study with a larger number of cantilevers to confirm this assumption. From the data obtained, the largest changes in frequency are in the order of tens of Hz. As seen in the initial testing stage, notch sizes of greater than 5 mm are sufficient to also change the frequency by tens of Hz. However, it is not necessarily desirable to modify blisk blades by such a large amount in reality, as the structure and aerodynamics of the blisk may be significantly compromised. The primary testing stage discussed in the next section provides an indication of the frequency changes for more realistic degrees of modification.

![Figure 11. Changes in frequency from baseline case obtained experimentally and computationally, mode 1.](image-url)
D. Primary Testing Stage

In this stage, a total of six blades for a single blisk were modified rather than individual blades in accordance with Castanier and Pierre’s (2002) s2 pattern. A FEM modal analysis was performed on each modified blisk, as well as the baseline blisk which had no modification. This stage allowed coupling effects between the blades via the disk to be included. As in the initial testing stage, notches were taken from the blade corners. Geometry of a notched research blisk is illustrated in Fig. 13 and the naming convention is presented in Table 2. The parameters varied for this test were the Δx and Δy dimensions; the angle of the notch stayed fixed at 45 degrees. The s2 pattern discussed in Castanier and Pierre’s (2002) study is reflected on the research blisk.

The blade was fixed in such a way as to reflect real clamping methods. It is important to note that while the clamping area had a significant effect on the blisk natural frequencies, the frequency trends remained the same.

Table 2. Research Blisk Nomenclature

<table>
<thead>
<tr>
<th>Δx, mm</th>
<th>Δy, mm</th>
<th>Designation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>Baseline</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>s2_1</td>
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<tr>
<td>5</td>
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<td>s2_5</td>
</tr>
</tbody>
</table>

Figure 12. Frequency for each test cantilever for a given notch size.

Figure 13. Research blisk notch geometry (left) and FEM clamping geometry (right).
A mesh independence study was conducted to determine a suitable mesh element size for the simulations. The results are presented in Fig. 14. It was noted that for mesh element sizes greater than 3 mm, the displacement distribution of the model was asymmetric. It is at present not known why this occurred, however future testing may use a blisk model generated using rotational symmetry and comparing the solutions. The computational software used was not able to solve for mesh elements sizes less than 1.5 mm. Computational time for a 1.5 mm element size mesh was significantly greater than that for a 2 mm element size mesh. For this reason a mesh size of 2 mm was used to ensure symmetry of the model, and find an accurate solution with a low computational cost. Results of the FEM analysis for the notched blisk are shown in Fig. 15. In comparison to the results of initial testing stage, only small changes in frequency were induced in the research blisk. However, notches were taken at 1 mm increments to better represent a realistic modification to blisk design. The notches taken in the cantilever reduced its length to by a greater proportion than the notches taken in the entire blisk; hence the frequency change for the research blisk is smaller.

At first it may appear that the notches in the research blisk do not change the frequency enough to counter the effect of the random mistuning seen in the experimental testing stage. However, it is expected that a single instance of random mistuning, such as a stone chip, would change the frequency of the research blisk much less than it would for a single cantilever. The frequency change caused by random mistuning in a blisk may only be in the order of single Hz. In this case, the notches are a sufficient form of intentional mistuning as they are able to offset this frequency change. Larger amounts of random mistuning may change the frequency by a greater degree. However, random mistuning causing such large changes often take the form of foreign object damage, or other serious defects. In this case it must be considered whether a blisk in this condition should be operated at all, and replacement may be the only viable option.

IV. Notes on the Results

Whilst the frequency trends seen in the initial testing stage and experimental testing stage are similar, the quantitative values are lower for the computational results. This may be due to uncertainties in the experimental testing stage, which are detailed here.

Figure 14. Results of a simple mesh independence study conducted on s2_5 (left) and an example of the asymmetric solution with a 4 mm mesh element size (right).

Figure 15. Changes in research blisk frequency from baseline case, mode 1.
Of primary concern is the method of clamping. Each cantilever was secured in a clamping block which could be tightened using four bolts. The clamping area of each cantilever was the same; lines scribed into the sample at 20 mm from the cantilever root could easily be aligned with the edge of the clamping block. However, it was difficult to achieve a uniform tightening across all bolts and thus hard to determine if the cantilever was clamped evenly. Non-uniform clamping may change the modes and natural frequencies excited between cantilevers, thus making comparison between them more difficult. The clamping block was secured to the workbench by two G-clamps, as seen in the image on the right in Fig. 9. This meant that the clamps had to be removed to change the cantilever being tested; it was difficult to ensure that the G-clamps were tightened by the same amount for each test. Variations in tightness of the G-clamps may result in damping of the oscillations in the cantilever as vibrations are transferred to the clamping block and workbench. This may produce an inaccurate reading of the natural frequencies of a given cantilever. In future it would be useful to develop a single clamping set-up which eliminates the need of both a clamping block and the workbench. This set-up should also allow uniform and repeatable clamping conditions to be achieved.

The method of excitation was also a considerable source of uncertainty. Each cantilever was flicked by a thumb nail to excite the oscillations. The initial amplitude of each excitation could not be controlled, and whilst this does not necessarily affect the natural frequencies of the cantilever the laser profilometer was not always able to detect the deformation of the cantilever over time for small amplitude excitations. This resulted in data that did not provide clear peaks making it difficult to ascertain the natural frequency for a given cantilever. In future, a more repeatable method of excitation should be employed, such as piezoelectric vibration excitation.

It would be useful to conduct additional tests with different laser profilometer settings to determine their effect on detecting the modal response of the cantilevers, and use an instrument that has a greater range of frequencies it can detect.

The primary testing stage results (see Fig. 15) followed a more linear trend that the initial and experimental testing results. This may be a result of the smaller range of notch sizes tested. It is expected that if a larger range of notch sizes is tested, the trend would be quadratic in shape due to the relationship between length and frequency in Eq. (4).

V. Concluding Remarks

From the results obtained in each testing stage, it is apparent that clipping the corners of blisk blades is a feasible method of intentional mistuning. It is important to note, however, that this is a rudimentary study using a narrow group of specifications. Whilst is provides an adequate foundation, there is considerable room for future investigation into this method; guidance for further study is provided in the following section.

VI. Recommendations

It is recommended that future studies conduct a more controlled and in-depth experimental testing stage in accordance with the details provided in Section VI. This will provide greater insight into quantifying random mistuning and the real-life behaviour of the cantilevers. The primary testing stage could be developed to include a greater range of notch sizes to identify if the frequency changes follow a quadratic shape as seen in the initial and experimental testing stages. Additionally, experiments could be conducted on the research blisk to quantify the effect of various degrees of random mistuning on its natural frequencies. An investigation into the forced response of the blisk and how this is affected by notch size should also be conducted. For the sake of completion, a rotating model could be tested to identify any effect this may have on the frequency change.

Future studies may be conducted on a realistic blisk model to investigate the aerodynamic effects of the modifications. This would determine if the benefit gained from frequency changes outweighs the aerodynamic implications of the notches.

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References


