Characterisation and Modelling of Steel Fibre Reinforced Reactive Powder Concrete

Mark Innes

The research and development of Reactive Powder Concrete has encompassed many areas of improvement, of which the poor ductility of Reactive Powder Concrete structural elements is a major concern. To improve the ductility of the concrete steel fibres have been added to the mix to provide reinforcement, but to also improve the ductility. Being a new area of development in engineering and construction, there has been a rapid growth in the amount of physical testing for the new age concrete. However, there has been limited development in the creation and adaptation of modelling techniques for fibre reinforced concrete. This report explores the availability of models and applies these to Reactive Powder Concrete. These results from the modelling process can then be validated by using the results in Ansys and comparison to physical results from practical testing in a laboratory.

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Secondly I would like to thanks Rachael Price, whom conducted the physical testing to accompany this report. Her help and support with me understanding the physical aspects of the RPC testing was greatly appreciated.

Finally I would like to thank my family and friends, who assisted me through the entirety of this project, giving needed support outside the realm of theoretical modelling techniques.
I. Introduction and background of Reactive Powder Concrete

In the concrete industry a material with brittle characteristics is an undesirable result. Brittle behaviour from a concrete structure can lead to catastrophic failure, which is a circumstance avoided in construction. Reactive Powder Concrete (RPC) is a product that exhibits these brittle characteristics. High Performance Concrete (HPC) is a material of low porosity with high cement and silica fume content, but is also void of any coarse aggregates (Malik, 2007). This high content of small sized particles gives HPC its dense characteristics, but also leads to the brittle behaviour. Other inclusions for the production of HPC include pulverised blast furnace slag or pulverised fly ash (Long et al, 2002). This brittle behaviour of HPC is overcome by the addition of small fibres (Ba et al, 2009) to create RPC. However, the strength exhibited by RPC is impressive, reaching 200 – 800MPa (RPC200 and RPC800) by using additional improvements such as pressure and heat treatment conditions during curing and setting (Zanni et al, 1996). A material that displays strength in this range is a desirable construction material; it is also environmentally friendly due to the reduction in use of coarse aggregates.

Though it is a new addition to the concrete family RPC has already shown promise in construction, being used for a footbridge in Sherbrooke, Quebec (Aitcin & Richard, 1996). This bridge was constructed using prefabricated elements, where the pressure and heat treatment was conducted by pouring the concrete into steel tubes and placing them in a steam room. An area of use for RPC was also explored outside of the construction arena, in the use of high-integrity containers (HIC) for the storage of nuclear wastes. The French nuclear industry explored the possibility due to the excellent microstructural and durability properties displayed by RPC. Along with the fibre reinforcement for improved ductility the concrete would have no problems supporting the structural load required when storing the nuclear wastes (Torrenti et al, 1996). These various applications discussed here show the versatile characteristic of RPC and the range of possibilities for future use.

The focus of this report will be to establish several modelling processes that can be employed to derive various characteristics of RPC. However, before researching modelling techniques, it needs to be determined what the input parameters will be and the desired outputs expected from these models. Once the necessary characteristics are established, the research can focus on determining necessary models and techniques that would derive the outcomes. The modelling processes used in this report include the basic modelling techniques, which will involve volumetric ratios and material properties, and a more accurate model was developed that uses equations developed by Christensen, 1979 and involve random orientation of fibres through a matrix. Other areas explored include the nonlocal theory of elasticity and the Mori-Tanaka theory, however, adaption of these models to RPC was difficult as will be discussed later in this report.

Once the theoretical models have been developed and the results generated, it is necessary to validate these results with comparison to physical testing, and the results obtained within. One set of comparisons were made with the results achieved by Bonneau et al, 1997. However, the major focus of this report is the verification of the models against the practical testing conducted by Price, 2009, whose research has focused on the physical characteristics of RPC in the areas of strength, rupture and elasticity. See Figure 1 - Test Specimens for Practical Testing, for the test specimens that were used by Price for the practical testing, the dimensions for the specimens are detailed below.
Test Specimen

<table>
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<tr>
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<th>Beam</th>
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<tr>
<td>Diameter = 75</td>
<td>Height = 100</td>
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<tr>
<td>Length = 160 Width = 60 Height = 60</td>
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</table>

Price uses fibres of length 13mm and a diameter of 0.15mm, in a concrete mix employing silica fume, fly ash and crushed quartz particles, full details on the mixes used will be detailed later in this report. The dimensions of the fibres employed are such due to the limitations of supply to the ADFA laboratory, the volumetric ratio used in physical testing was 5% volumetric ratio, as Price has assumed this to be the optimum fibre content ratio. Practical testing into the changes that occur to the concrete due to heat and pressure treatment, as well as testing RPC with and without fibre reinforcement, was also conducted. An example mix design as used by Price is shown here in Table 1 - Standard Mix Design (Price, 2009).

| Table 1 - Standard Mix Design (Price, 2009) |
|-------------------------------|----------------|
| Material                       | Amount         |
| Cement                         | 600 kg         |
| Water                          | 150 kg         |
| Silica Fume                    | 240 L          |
| Super plasticiser              | 15 L           |
| Finely Ground Sand             | 870 kg         |
| Quartz Powder                  | 570 kg         |
| Fly Ash                        | 250 kg         |
| Steel Fibres                   | 275 kg         |
| **Expected Density**           | **2400 kg/m³** |

The design shown in table 1 is the amounts of each ingredient that would be required to develop 1 cubic meter of RPC. However, during the practical testing it is not necessary to generate this much RPC at any one time, so the amounts are scaled to represent the amount that would be needed to make the test specimens required for each batch. The models mentioned earlier will be used to mirror the mix designs employed by Price to get the most accurate comparison that is possible. With results from each of the research models, these results can be verified and validated for each, drawing relevant conclusions from each. With these standard tests completed this report will look into how alterations to the RPC mix affect the various characteristics. Such changes include the use of different fibres (glass and natural fibres), as well as variations to the volumetric ratio of fibres.

II. Literature Review

A. Reactive Powder Concrete

Since the initial development of RPC there has been a range of research conducted on the mechanical properties, mix designs and microstructure of the concrete with fibre reinforcements. Tests conducted on the tensile properties of RPC (Behloul et al, 1996) concluded that RPC failed in a brittle manner when tested. Even though this brittle behaviour is evident, compressive strengths have been achieved that give great weight to the argument for the use of RPC as a construction material, up to 810MPa (Richard, 1996). When fibres are added to a mix it is important to analyse the interface that exists between the fibre and the matrix, and how they interact during strength testing. The high density matrix of RPC is a property that exhibits excellent pullout strength for this interface between the fibre and the matrix (Yang et al, 2007); this high strength bond between concrete and fibre is an excellent characteristic as any microcracks that may occur through the matrix will be easily stopped when they encounter the fibre. These fibre interactions also begin to give understanding to the optimum fibre content that is necessary for the most economic, but structurally sound RPC. Another focus area for the improvement of RPC is the heat treatment and the application of pressure to the concrete after setting (Richard & Cheyrezy, 1995), this process was responsible for the extremely high strength concretes that have been created thus far.

The manufacture of RPC, using very small particles rather than traditional coarse aggregates that are present in standard concrete, has led to an increase in cement content from 300-500kg/m³ up to around 900-1000kg/m³ (Collepardi et al, 1997), for this reason people have attempted to develop alternative mix designs than those originally used for the production of RPC. The method employed by Collepardi involved the addition of some coarse aggregates in the attempt to limit the shrinkage and creep strain brought on by standard RPC mixes. Further research has also been done into optimising the properties and durability of RPC (Chaheen & Shrive,
also, the pressure and heat treatment was conducted by curing the RPC inside steel casings. The areas for research into the improvement of RPC are nearly limitless, being a new product into the construction industry interest levels are high giving inspiration for continual improvement of RPC. Physical testing of RPC has shown it has a high specific weight due to the pressure treatment to fresh RPC, getting values up to 3000kg/m³. This may be a short coming of RPC, with an increase in the silica fume content the compressive strength was not affected, however the density was reduced to as low as 1900kg/m³ (Sadrekarimi, 2004). This is only one example of many available of the range of investigations being conducted on the physical properties of RPC.

This information gives a good understanding of the composition and physical characteristics of RPC, however, this report is directed towards the modelling and analysis of RPC, it therefore paramount that research is conducted into the modelling techniques available and the results or outcomes of these tests be analysed. By doing this it will give insight into the possible techniques that may be employed to theoretically determine the characteristics of RPC.

B. Modelling Techniques

When modelling RPC it cannot be treated as an isotropic material (a material that can be simply analysed, with basic equations), so the RPC material will be treated as a quasi-isotropic material, that is, the material has approximated isotropy by orientating the various materials in the mix to a single plane (UK Encyclopaedia, 2009). This is a focus for the Orientation Model discussed later, however, the Basic Models are lacking in the use of angles and standard axes when calculating equivalent properties. The mechanical properties of fibre reinforced composites are affected by a number of parameters including fibre length, geometry, orientation, dispersion and adhesion between the fibre and matrix (Kalaiprasad et al, 1997). This research by Kalaiprasad looks at the reinforcement of composite materials, however, has been adapted to suit the reinforcement of RPC by steel fibres. As they are basic models the change from composite materials to concrete is assumed to have a negligible effect on the results. When modelling RPC it is important to remember that the transfer of load between the fibres and the surrounding matrix is a function of the mechanical characteristics of each component, the percentage of fibres and the local orientation of the fibre reinforcement (Bayard and Ple, 2003). This report is aiming to produce an analytical representation of the fibre phase properties, matrix phase properties as well as the volumetric concentration of each (Christensen, 1979).

While researching likely prospects for the modelling of RPC there were many models found that have been developed by various agencies, however, to reconstruct some of these models would take time and resources that are not available to this project. The mass-spring-damper system (Fujikake et al, 2006) looked at the impact response of RPC with both static and dynamic loading, discovering that the flexural damage is not related to the load type, but in fact the amount of deformation. The modelling of RPC in tension was characterised by the 1-D Rheological Model for RPC (Kovacs et al, 1998), which used a set up of springs (with relative stiffness and friction strengths) to determine the mechanical characteristics of RPC. However, this focussed on the stresses in the fibre and matrix separately, before determining the point of cracking within the concrete, and the stress and strain associated with cracking points. To validate the modelling techniques often requires comparison to previous results from other researchers, or to physical results carried out during laboratory experiments. Shen and Li, 2002, developed micromechanics models which were then used in conjunction with finite element models to establish strain and strength parameters for validation of the models created. This process is similar to that used by this report, where the material properties developed were used in conjunction with Ansys to develop finite element analysis models. This research also involved parameters like those used in this report of $E_{11}$, $V_{12}$, and $K_{12}$, these will be discussed in greater detail later in this report when discussing the Orientation Model.

The Nonlocal Theory of Elasticity takes the interaction between atoms into account, therefore the stresses and strains at a single point are affected by all points throughout the body (Povstenko, 1999). This modelling technique was originally proposed by Eringen (Wang et al, 2008), and has been employed within various areas to determine parameters of elements that otherwise require assumptions to be made. This modelling technique also involves the use of Lame constants (μ and λ) as discussed later in this report (Polizotto, 2001). Also employed in the nonlocal theory are ‘kernels’, these kernels have been derived analytically, based on the structure of the material at the atomic scale (Picu, 2002). The use of nonlocal theory for analysis has been widely used in the analysis of Carbon Nanotubes (CNT) that has been used in composite reinforcement.

Since the introduction of CNT in 1991, there has been an ever increasing rise in the amount of research conducted, considering the limited availability of molecular dynamics simulation (Yan et al, 2008). As CNT are so small in size they are difficult to model, nonlocal analysis for CNT has focussed on developing material properties of a single tube with the use of nonlocal elasticity rod theories (Wang et al, 2008) and the like. Other models for the development of material properties of CNT include the Force Field Model, Bond Order Model and the Semi-empirical Model (Ruff et al, 2003). Micromechanics analysis of CNT has also been conducted
looking at getting various strength parameters for CNT reinforced composites (Kim et al, 2009) which also employs the Halpin-Tsai model, as does this report. The early assumption of this project was that a modelling technique for a fibre reinforced composite could be obtained and altered so that RPC could be modelled, apart from the basic modelling techniques employed this was found to be lacking as discussed here.

The research by Goh et al, 2004 analysed the plastic stress transfer and fibre pullout strength (among other properties) of short-fibre composite materials. However, the modelling used was more focussed on the finite element analysis of the composites, and the equations employed looked at the reinforcement on a single plane (like the basic modelling) and so was not applicable to the random orientation of the reinforcement of RPC. The Eshelby’s tensor for a circular cylindrical inclusion with weakened interface bonding was adopted for the modelling of continuous fibres (Lee & Pyo, 2009). This modelling technique is very detailed and uses Eshelby’s tensor (similar to the process for the establishment of the Orientation Model equations), however, is focused on the model of ‘continuous’ fibres with imperfect interfaces. The modelling of RPC needs to look at the reinforcement by finite length fibres, with a strong interface bond. More research in the response of Composite Materials was conducted looking at their performance in a vehicle crash (Selvarajalu, 2003), where detailed analysis was conducted on the impact response of composite materials. This report also looks at the use of the Mori-Tanaka model; unfortunately this model looks at materials with a high volumetric fraction of fibre reinforcement (30-40%), where RPC only employs 3-7% of fibre volume.

When modelling a matrix that has been reinforced with small fibres the interface between the two materials is an important area for analysis, as any debonding that occurs or the like will have an impact on the overall properties of the final product. This is an area that has become more apparent in the development of composite materials, where interface mechanics are a growing area of research (Pan & Takeda, 1998). This comparison between RPC and composite materials lead to the investigation of CNT reinforced composites by the use of other techniques. One such method was to investigate the CNT down to the molecular level and explore the macroscopic properties of CNT composites from the elastic deformation of a representative volume element (Hu et al, 2005). Other methods utilise beam models such as the nonlocal Euler or Timoshenko beam theories (Lu et al, 2007 and Heireche et al, 2007) treating a nanotube as a rod to determine the material properties of the CNT. These material properties could then be incorporated into a finite element analysis program, developing a mesh of the CNT to then analyse the physical response. A process such as this is possible for steel fibre reinforcement within an RPC matrix; however, it would only focus on a single steel fibre, of which the material properties are already known. Also, do delve into the molecular structure of RPC would be too detailed for the purposes of this report. However, one area gained from these reports was the possibility of finite element analysis after determining the material properties of RPC. This will allow for analysis of structural members or even the test specimens used in the lab to get a more detailed comparison.

### III. Explanation of models analysed

#### A. Desired Input Parameters and Expected Outputs

1. **Standard Characteristics and Inputs**

   The inputs for these modelling techniques to be analysed need to be easily visible, meaning they must be obvious inputs that can be varied easily. For the basic modelling the Series and Parallel models need only the simplest of characteristics, being the Young’s Modulus of the fibres and concrete, as well as the volumetric ratio for each. These are simple equations, and are easily to manipulate, though the range of outputs is limited to only an equivalent Young’s Modulus. The more detailed models require further inputs, needing more focus on the orientation and distribution of the fibres, therefore, incorporating extra ‘equivalent’ properties for the full analysis of RPC.

2. **Desired Outputs**

   The major focus for output criteria will be the equivalent Young’s Modulus and Poisson’s Ratio, an ideal scenario would be to utilize a fully established model that models not only the characteristics of the RPC but also the strength characteristics. Through the research discussed earlier, there were limited modelling techniques available that focussed solely on the stress and strain. The Orientation Model uses these characteristics to derive the material properties, but the basic models are found lacking. The Mori-Tanaka theory also uses the stress and strain in areas, but due to reasons discussed later was not available to the research area of this report. This said, obtaining the material characteristics allows for further analysis to be conducted focussing on more detailed modelling looking at the Shear and Bulk Modulus (G and K respectively), not only this, but the material properties can be used in a program such as Ansys to conduct modelling on various structures or elements that may be employed in the construction industry or the testing lab.
B. Problems faced

In metal matrix modelling a simple linear elastic model such as Hooke’s Law suffices for the fibre’s mechanical methodology, while the matrix (a metal alloy) will be far more rate and temperature dependant (Ellyin and Xia, 2001). However, this assumption is not possible for the random orientation that is present within RPC, due to the different natures and characteristics of the metal and the concrete matrices, as well as the process undertaken to develop the two fibre reinforced bodies. Concrete requires greater amounts of mixing, in turn generating a random and hopefully uniform distribution of the fibres.

Due to the limited availability, the sand and inclusions for the production of the RPC specimens was lacking. To overcome the problem of the sand, standard river sand was crushed in a ball mill to achieve a sand with an extremely fine particle size in the micro scale (μm). During physical tests it would have been desirable to perform tests on specimens with varying fibre lengths and dimensions to research the changes that occur, however, this was not possible due to reasons already mentioned. Another area of concern for the fibres is lumping. When the RPC is mixed in its wet form the slight fibres are easily bent and warped by the mixing implements, this causes the fibres to catch together creating small clumps of fibres through the mix. A good indication of the size of the fibres compared to the concrete test specimens is shown in Figure 2 - Steel Fibres in RPC Cylinder.

The addition of fibres into RPC can provide some areas for concern when mixing the wet concrete as the fibres can lump into a single point, not giving uniform distribution which is desired. When the fibres are added to the mix they are bent and warped due to the impact of the mixing instruments, allowing the fibres to catch together easily which causes lumping. Only 5% of the volume is made up by the fibres, if they are lumped into one location then the rest of the test element will be just RPC and have no reinforcement, as shown in Figure 3 - Lumped Fibres. In this figure it is shown that a large portion of the fibres have in this case settled/lumped into a small area of the concrete matrix. During the mixing process care needs to be taken to ensure that the fibres are uniformly distributed so assuming the fibres have not lumped remains accurate. The bending and warping can also be advantageous, in that it can improve the pull out strength of the fibre (so long as the entire fibre is pulled out), but too much deformation can lead to the fibre fracturing or the matrix splitting (Banthia, 1990). To help limit the impact of the problems faced, assumptions can be made to provide greater analysis and for ease of understanding.

There are methods for improving the toughness of high performance concrete (which includes RPC) one is the addition of steel fibres into the mix as discussed by this report another is the application of pressure to the
concrete element during setting which will increase density, reduce entrapped air and removes the excess water from the mix. This pressure application will assist in increasing the compressive strength of the concrete, which has reached as high as 810MPa in previous developments (Richard, 1996). The compressive strength can also be increased by the application of heat during setting or curing. The treatment can be done post-set by applying 90°C, which will increase the rate of the pozzolanic reaction and modify the microstructure of the hydrates formed. It can also be applied at a high heat which greatly dehydrates the hardened paste, but also leads to the formation of crystalline hydrates known as Xonotlites (Richard, 1996).

The removal of any free water from the concrete mix is also a favourable outcome, as free water in a concrete mix can allow aggressive agents to infiltrate and degrade the concrete (Cheyrezy, et al, 1995). This degradation of concrete can lead to poor durability and ultimately the requirements for maintenance and overall costs will be higher. The heat treatment of RPC leads to an increased rate of pozzolanic reaction, meaning the bound water to total water content will be closer to 100% (no free water in the mix). Figure 4 - Water Content due to Heat Treatment shows how the heat treatment affects the bound water to total water content ratio, while there is also an increase in the pozzolanic reaction due to heat treatment, both of which are desirable outcomes. It can also be seen from Figure 4 that the pressed RPC at certain temperatures also displays an increase, implying that the addition of pressure can also have an advantageous effect. To provide maximum improvement to RPC both of these methods can be employed at the same time so that a maximum compressive strength can be achieved for the concrete.

It is for these reasons that Price, 2009 is attempting to mirror the application of heat and pressure to RPC whilst curing the concrete. By achieving higher strength concrete in the lab for RPC and then comparing this strength against the ductility of the product will give understanding into the further use of RPC in the construction industry, similar to the projects as discussed previously in this report.

However, by looking at the modelling techniques that are used in this report it is clear to see that nowhere in any of the calculations is there any inclusion of heat treatment or pressure application during curing. It is therefore an assumption of all the modelling techniques that the concrete-fibre mix is just that, without any extra work being conducted to improve the final material properties. This discrepancy may be the cause for some of the variations in the results when they were verified against practical testing. The tests with pressure and heat treatment are expected to yield higher values, than those which receive no such treatment. Therefore, when making comparison, as done within this report, it is vital to ensure that notification is provided detailing any further treatment the concrete has received so as not to provide misleading results.

C. Assumptions

The modelling of RPC can be a simple process if accurate assumptions are made about the mix. For example the series and parallel models assume that the fibres are of a continuous length and all fibres are of an identical orientation. The Hirsch Model uses a combination of the series and parallel models, therefore assuming only two directions of orientation for the fibres. To achieve a more detailed analysis of more characteristics need to be determined regarding the fibre and the matrix. The Orientation Model incorporates this detailed by creating composite cylinders, of which assumptions are made about the material properties of these fibre/concrete cylinders. This information is then used to calculate overall properties for the body being tested by giving a random orientation to the cylinders in three different planes.
When considering the practical testing conducted and making comparisons to the test results, it is assumed that the fibres during mixing are uniformly distributed throughout the entire mix, and that no lumping has occurred. The modelling techniques also assume that the fibres remain completely straight during all stages of testing, which is nearly impossible to achieve in practice unless an unreasonable amount of time and care is taken during the mixing phases. With the modelling techniques employed that only provide the user with material characteristics, and so when tested with Ansys to develop strength parameters, it is assumed that all RPC developed after would be of the same mix design and creation process as that employed by Price, 2009, unless further analysis was done on alternate mix designs and physical tests. With this understanding of the problems that were faced and various assumptions to limit the effect of the problem areas, each of the modelling techniques which were researched will now be mentioned in detail to better know the processes involved to develop the required results from each.

D. Basic Models

When researching the modelling techniques available a report detailed a number of techniques that are all assumed to be of a ‘basic’ nature (Kalaprasad et al, 1997). However, some versions of the equations have been developed into greater detail so that they now incorporate more characteristics into the modelling process with the expected outcome being more accurate theoretical results. The use of basic modelling by this report looks at setting a bench mark value for the models which incorporate more detail (orientation, finite fibre lengths, etc). The more modelling techniques that are employed the more validation that can be carried out on individual models, as well as comparisons to the practical results achieved in the lab. Detailed in equation 1 (Series Model) and equation 2 (Parallel Model) are two of the very basic forms of modelling that are available, taking into account only Young’s Modulus and the volumetric ratio of both the matrix and the fibre.

\[
E = E_f V_f + E_m V_m \tag{1}
\]

\[
E = \frac{E_f E_m}{(E_m V_f + E_f V_m)} \tag{2}
\]

Constant values used in the above equations are detailed here;

- \(E_m\) = Young’s Modulus of the Matrix
- \(E_f\) = Young’s Modulus of the Fibre
- \(V_m\) = Volumetric Ratio of Matrix
- \(V_f\) = Volumetric Ratio of Fibres

![Graphical Representation of Basic Models](image_url)
The equation developed by Hirsch, and the Series and Parallel Models are shown in Figure 5 - Graphical Representation of Basic Models. This image depicting the basic modelling techniques indicates that the series model assumes the fibres are acting perpendicular to the direction of stress, while the parallel model orientates the fibres in the direction of the stress. Also, both models assume that the fibres are of continuous length throughout the matrix in which they are suspended. While Hirsch’s model is a combination of the Series and Parallel models, however, it is still a basic equation as the orientation and location of the fibres are not considered. The equation developed by Hirsch (equation 3) is detailed here.

\[ E = x(\frac{E_m V_m}{E_f V_f}) + (1-x) \frac{E_f E_m}{E_m V_f + E_f V_m} \]  

The variable \( x \) included in this equation is related to the stress transfer between the fibre and the matrix, where fibre orientation was assumed to be the major factor that is affecting the value of \( x \) (Kalaprasad et al, 1997). Kalaprasad found agreement to occur between theoretical and physical results to occurred when \( x=0.4 \), giving the result shown in equation 4.

\[ E = 0.4 \left( \frac{E_m V_m}{E_f V_f} + 0.6 \frac{E_f E_m}{E_m V_f + E_f V_m} \right) \]  

The use of \( x=0.4 \) is a simplistic way of saying that 40% of the fibres are orientated in a series formation (equation 1), while the other 60% are in parallel (equation 2). However, due to the random orientation that occurs during the mixing of RPC it is not possible to compare the value of 0.4 (obtained for a composite material) to the nature of orientation within RPC. Therefore it is a requirement of this report to determine a value of \( x \) that is suitable for use when analysing RPC. For this calculation we take the use of the experimental results achieved by Price, and determine a new value for \( x \). Using the results shown in Table 4 – Experimental Results, Price 2009, it is plausible to develop a value for the \( x \) parameter in the Hirsch equation relevant to RPC, see equation 5. The values used for this calculation include an RPC Young’s Modulus of 35.1 GPa (batches 1 and 2), with the Young’s Modulus of the matrix (unfibred concrete) as 31.9 GPa. The values for steel are \( E = 200 \) GPa, with a volumetric ratio of 5%.

\[ 35.1 = x(31.9 \ast 0.95 + 200 \ast 0.05) + (1-x) \ast \frac{200 \ast 31.9}{31.9 \ast 0.05 + 200 \ast 0.95} \]

Solving this equation for \( x \) yields the value of:

\[ x = 0.257 \]

Further discussion of the Hirsch model is detailed later in this report when covering comparison of the models and validation and verification of the results. During the research into the basic modelling equations, it was discovered that one variation of the simplistic approach encompassed a more detailed and accurate analysis. Detail was added by incorporating fibre geometry and orientation into the equation, this was the Halpin-Tsai model, shown in equation 6.

\[ E = \frac{E_m (1 + A \eta V_f)}{(1 - \eta V_f)} \]  

Where:

\[ \eta = \frac{\left(\frac{V_f}{E_m}\right)^{-1}}{\left(\frac{E_f}{V_f} + \frac{E_m}{A}\right)} \]  

\( A = \) measure of fibre geometry and fibre loading conditions

The Halpin-Tsai relationship has been modified to provide more relationship between the fibre and matrix, these results are still very basic, but provide a decent level of analysis for comparison between models and validation with practical results.
For the calculations of \( A \) used in the Halpin-Tsai equation the following was used from Halpin and Kardos, 1976, detailed in equation 7.

\[
A = \frac{(k_m + m_m)A_m}{(k_f + m_f)A_f}
\]  

(7)

Where:

- \( k_{m/f} \) = Bulk Modulus of the matrix/fibre
- \( m_{m/f} \) = Shear Modulus of the matrix/fibre
- \( A_{m/f} \) = Area of the matrix or the fibre

For the calculation of the area values the matrix was assumed to be a 13x13mm square, while the fibres were taken to be slender rectangles of dimensions 13x0.15mm. Only the area of a single plane is needed (one square of concrete, and a single fibre) the volumetric ratios are taken into account during the earlier equations.

E. Orientation Model

Unless a large amount of care was taken it can be extremely difficult to ‘perfectly’ distribute the fibres through the RPC mix, where in reality the fibres tend to distribute themselves within small clumps or groups of fibres with similar orientation (Bayard and Ple, 2003). It can therefore be assumed that instead of modelling a single fibre or numerous single fibres within a volume element, these clumps of fibres (on the same orientation) can be modelled instead. One method that looks at this is the process known as the Orientation Model; this model was developed by Bayard and Ple using equations developed by Christensen, 1979. This report focuses on a similar process, however, uses the aid of Christensen to develop another set of equations and results as discussed hereunder. There were three areas of modelling developed; three dimensional and two dimensional calculations, as well as asymptotic predictions.

The version of equations used in this report does not employ groups of fibres, instead encases them in a concrete cylinder to create what is known as a composite cylinder, as shown here in Figure 6 - Composite Cylinder orientated on three way axis.

![Figure 6 - Composite Cylinder orientated on three way axis](image)

In this figure the dashed line represents what will become the ‘composite cylinder’ for the modelling purposes. Before determining the properties of the entire matrix, the effective properties of the cylinder is determined, as shown below. The simple form for the effective modulus \( E_{11} \) is in equation 8.

\[
E_{11} = \frac{< \sigma_{zz} >}{\varepsilon}
\]

(8)

The transformation of equation 8 yields the result shown in equation 9.
\[ E_{11} = \frac{1}{\pi b^2 \varepsilon} \int_A \sigma_z(r) dA \]  

(9)

Where;

- \( A \) = Cross-sectional area
- \( b \) = Radius of Fibre and Surrounding Concrete
- \( \varepsilon \) = Strain imposed on fibre
- \( \sigma_z \) = Related stress on fibre

The performance of the double integral in equation 9, leads to the establishment of equation 10. Here the effective modulus for the composite cylinders is reproduced in a form that uses simple variables, and the equations can be simply solved with a mathematical program.

\[ E_{11} = cE_f + (1 - c)E_m + \frac{4c(1 - c)(v_f - v_m)^2 \mu_m}{\frac{(1 - c)\mu_m}{k_f + \frac{\mu_f}{3}} + \frac{c\mu_m}{k_m + \frac{\mu_m}{3}} + 1} \]  

(10)

Where;

- \( c \) = Volumetric Fibre Ratio (\( V_{\text{inclusions}}/V_{\text{matrix}} \))
- \( E_f \) = Young’s Modulus of the Fibre
- \( E_m \) = Young’s Modulus of the Matrix
- \( v_f \) = Poisson’s Ratio of Fibres
- \( v_m \) = Poisson’s Ratio of Matrix
- \( \mu_f \) = Shear Modulus of Fibres
- \( \mu_m \) = Shear Modulus of Matrix
- \( k_f \) = Bulk Modulus of Fibres
- \( k_m \) = Bulk Modulus of Matrix

The property of Effective Modulus (\( E_{11} \)) is also known as the uniaxial modulus (Christensen, 1979). The effective Poisson’s Ratio (in the direction of the fibre) can be established with a similar process to that used for the establishment of the effective modulus, as shown in equation 11.

\[ \nu_{12} = cv_f + (1 - c)v_m + \frac{c(1 - c)(v_f - v_m) \left[ \frac{\mu_m}{(k_m + \frac{\mu_m}{3})} - \frac{\mu_m}{(k_f + \frac{\mu_f}{3})} \right]}{\frac{(1 - c)\mu_m}{k_f + \frac{\mu_f}{3}} + \frac{c\mu_m}{k_m + \frac{\mu_m}{3}} + 1} \]  

(11)

The solving of equation 11 uses the ratio detailed here in equation 12.

\[ \nu_{12} = -\frac{u_r}{\varepsilon b} \]  

(12)

In which;

- \( u_r \) = Displacement Field

The Plane Strain Bulk Modulus (equation 13) and the Shear Modulus (equation 14) can be established in similar fashion to those detailed in \( E_{11} \) and \( \nu_{12} \), when using the displacement fields provided by Christensen the following equations are established.
\[ K_{23} = k_m + \frac{\mu_m}{3} + \frac{c}{k_f - k_m + (\frac{\mu_f - \mu_m}{3})} + \frac{(1 - c)}{k_m + (\frac{4\mu_m}{3})} \]  
\[ \mu_{12} = \frac{\mu_f (1 + c) + \mu_m (1 - c)}{\mu_f (1 - c) + \mu_m (1 + c)} \mu_m \]  

However, before effective properties can be established for three dimensional or two dimensional cases, the transverse shear \((\mu_{23})\) of the fibre system needs to be considered. The transverse shear is a characteristic that varies with the two and three dimensional cases, due to the different orientations that are needed for the two cases. The process involved for the development of the transverse shear model for the composite cylinder uses what is known as Eshelby’s Formula. Christensen provides a solution for the transverse shear as depicted in equation 15.

\[ \frac{\mu_{23}}{\mu_m} = 1 + \frac{c}{\mu_m/(\mu_f - \mu_m) + (k_m + \frac{7}{3}\mu_m)/(2k_m + \frac{8}{3}\mu_m)} \]  

However when considering the transverse shear the volumetric ratio \((c)\) takes on the form shown in equation 16.

\[ c = \left(\frac{a}{b}\right)^2 \]  

This ratio between the two radii used to determine the ratio ‘c’ needs to remain constant for all situations no matter the variation in the radius of the fibre. In the case RPC the steel fibres have a constant diameter of 0.15mm, and so this allows for the single dimension for the constant ‘b’. As shown in Figure 7 - Dimensions of a and b for Transverse Shear Calculation, the image depicts the end of a composite cylinder is depicted giving the dimensions for a and b. For this report the diameter of the fibres used is known, however, the concrete surrounds need to be assumed, the following values were established, \(a = 0.075\text{mm}\) and \(b = 0.225\text{mm}\).
The modelling of the cube with scattered composite cylinders would be expected to be most accurately modelled if two orientation angles are employed. This is where the three dimensional model is accurate, however, with extra analysis comes more areas for convoluted analyses when developing the necessary equations. As shown in the equation below the stress-strain ratio is analysed with the two angles theta (θ) and phi (φ). The process involved for the determination of the relevant characteristics of the media requires a body of similarly aligned fibres to have a strain condition assigned to these fibres (Christensen, 1979). By allowing for this condition it removes the need for trying to assign boundary conditions to the representative volume element, containing the distributed fibres. In order to achieve the results the following must occur; initially the double integral as shown in equation 17, must be solved.

\[
\left. \frac{\sigma'_{ij}}{\varepsilon'_{33}} \right|_{\text{random}} = \frac{1}{2\pi} \int_0^{\pi} \frac{\sigma'_{ij}}{\varepsilon'_{33}} \sin \theta \, d\theta \, d\phi
\]  

The solving of the three dimensional case involves the use of five constants \(C_{11}, C_{12}, C_{22}, C_{23} \text{ and } C_{66}\) which are used in generating two engineering properties of the RPC which are the shear (equation 18) and bulk (equation 19) moduli; \(\mu\) and \(k\) respectively.

\[
\mu = \frac{1}{15} \times (E_{11} + (1 - 2 \times v_{12})^2 \times K_{23} + 6 \times (\mu_{12} + \mu_{23}))  
\]

\[
k = \frac{1}{9} (E_{11} + 4 \times (1 + v_{12})^2 \times K_{23})  
\]

These can then be directly related the Young’s Modulus and Poisson’s Ratio (equations 20 and 21 respectively) through the standard equations as detailed here.

\[
E_{3D} = \frac{9k\mu}{3k + \mu}  
\]

\[
v_{3D} = \frac{3k - 2\mu}{2(3k + \mu)}  
\]

With the properties of the three dimensional model known, this report moves into the development of a model that incorporates only a two dimensional understanding of the representative volume element. This modelling process looks at the plane stress conditions rather than the strain elements explored for the development of the three dimensional model. To obtain the two dimensional model the following integral in equation 22 needs to be solved.

\[
\left. \frac{\sigma'_{ij}}{\varepsilon'_{11}} \right|_{\text{random}} = \int_0^{\pi} \frac{\sigma'_{ij}}{\varepsilon'_{11}} \, d\theta
\]
This situation is only looking at two planes of interest (two dimensional) and again uses the five constants ($C_{11}$ to $C_{66}$) to develop four extra constants being $Q_{11}$, $Q_{12}$, $Q_{22}$ and $Q_{66}$. These are then used to establish two stress strain ratios, which are derived from assuming the only non-zero strain is in the direction of the fibre, along axis 1. Here, the planar isotropic properties, equation 23 are utilised.

$$\frac{E}{1 - \nu^2} \text{ and } \frac{vE}{1 - \nu^2}$$ (23)

These relationships between the Young’s Modulus and the Poisson’s Ratio allow for the development of an equation for each including the newly established ‘Q’ constants, which in turn yields equations 24 and 25 shown below for the development of the material properties pertaining to the two dimensional case.

$$E = \frac{u_1^2 + u_2^2}{u_1}$$ (24)

$$\nu = \frac{u_2}{u_1}$$ (25)

For complete development of $E$ and $\nu$ the use of equations 26 and 27 are used to complete the modelling process.

$$u_1 = \frac{3}{8} E_{11} + \frac{\mu_{12}}{2} + \frac{(3 + 2\nu_{12} + 3\nu_2^2)\mu_{23}K_{23}}{2(\mu_{23} + K_{23})}$$ (26)

$$u_2 = \frac{1}{8} E_{11} - \frac{\mu_{12}}{2} + \frac{(1 + 6\nu_{12} + \nu_2^2)\mu_{23}K_{23}}{2(\mu_{23} + K_{23})}$$ (27)

The three and two dimensional models just discussed require very detailed analysis and comprehensive calculations to establish the results shown in equation 6 through to equation 18. For this reason asymptotic predictions have been developed using bounds set out by Christensen, to give the following results for a matrix body with fibre reinforcement (equations 28 through to 32).

$$E = cE_f + (1 - c)E_m + 4c(1 - c)\mu_m \left[ \frac{(v_f - v_m)^2}{(1 - c)\mu_m} + \frac{c\mu_m}{(k_f + \mu_m/3) + 1} \right]$$ (28)

$$\nu_{12} = \frac{c(1 - c)(v_f - v_m)}{(k_f + \mu_m/3)} + \frac{c\mu_m}{(k_m + \mu_m/3) + 1}$$ (29)

$$K_{23} = k_m + \frac{\mu_m}{3} + \frac{c}{k_f - k_m + \frac{(\mu_f - \mu_m)/3}{k_m + (4\mu_m)/3}} + \frac{(1 - c)}{k_m + (4\mu_m)/3}$$ (30)
F. Nonlocal Theory of Elasticity

The nonlocal theory of elasticity (NTE) was first utilised by Eringen from 1976-1983 (Wang et al, 2008). The NTE looks at a point \( x \) and the stress at this point is a function of the strain field at every point in the body, not just the neighbouring points (Yan, et al, 2007), see Figure 9 - Nonlocal Elasticity Theory for the graphical representation. Here in this figure the point \( x \) (black cube) is affected by not only the points neighbouring the reference point, but all points through the beam (white with black outline). This theory has been heavily used in the research of CNT reinforcement and the assessment of the mechanical properties of CNT.

This connection that exists between the stress and strain through the entire body can be represented in the form of equation 33, from Kuliev, 1979.

\[
\sigma_{ij} = \lambda \varepsilon_{rr} \delta_{ij} + 2 \mu \varepsilon_{ij} + \int_{D=F} \left( \lambda_i \varepsilon_{rr} \delta_{ij} + 2 \mu_i \varepsilon_{ij} \right) d\nu \tag{33}
\]

Where;

\[ \mu_1 & \lambda_i = \text{Lame Constants} \]

These Lame Constants represent the moduli of nonlocal elasticity, while the integral in the equations is evaluated over the deformed volume \( F \). The Lame Constants are functions of the temperature and are used in an attempt to moderate the effect of the environment. However, this version of the NTE is extremely dated and there have been many attempts to recreate the original equation proposed by Eringen. As mentioned earlier there have been many attempts to generate the adaptations for the use in the modelling of CNT reinforced composites, as well as other improvements. One such report (Sciarra, 2009) has looked into the used of the NTE to develop a model that looks at a two phase media, including impacts from the thermodynamic framework and also attempted to create an interface between the NTE and finite element analysis.

There has also been research into incorporating other models in the NTE to better represent the necessary material being modelled. Such examples include the incorporation of the Timoshenko beam theory (Heireche, et al, 2008) and the nonlocal Euler-Bernoulli theory (Yan, et al, 2007). However, these models look at the inclusion as a single rod within a media and develop only the material properties for the inclusion, not the effective characteristics for the representative volume element. The process developed by Yan, looks into the Euler-Bernoulli Beam Theory, where the bending moments and coordinate systems of a CNT are analysed, before including the moment of inertia and the transverse deflection of the CNT within the body. Here the CNT analysed include multiple walls, and rather than the analysis of the CNT subject to static loads, the vibration analysis is conducted on a CNT that is assumed to be simply supported.

Further research into the understanding and use of the NTE on RPC was conducted but similar results were encountered regarding CNT analysis. It was hoped that some research had been conducted on the steel fibre reinforcement of RPC with the NTE; however, none was available at the time of this report. Another problem with the NTE was the fact that many of the models only focussed on a single plane (such as the simply supported CNT), where the fibres within RPC are scattered randomly throughout the concrete with a range of different orientations. For these reasons and the lack of a sufficient modelling technique the use of the NTE for
the modelling of RPC has been discontinued, another possible modelling technique for use on RPC is the Mori-Tanaka theory.

G. Mori-Tanaka Theory

The Mori-Tanaka model can be implemented with materials that have multiphase particles (Tan, et al, 2005), but is also highly focused on materials with high concentration of particles within the volume. The focus of the model is on interface debonding being the greater area for concern, governing the behaviour of the material. This process also uses a representative volume element (RVE) of the matrix material and particle inclusions. The very basic forms of the equations in this technique show a comparison between particle volume fractions and the stress-strain. Here equation 34 depicts the relationship of the average stress to the stress in the matrix, the particles (fibre reinforcement) and the volumetric fraction of each.

$$\bar{\sigma} = (1 - f)\sigma^m + f\sigma^p$$ (34)

In which:

- \(\bar{\sigma}\) = Average Stress
- \(f\) = Fibre/particle volumetric fraction
- \(\sigma^m\) = Stress in Matrix
- \(\sigma^p\) = Stress in Particle

The average strain is related in a similar fashion, however now equation 35 includes the effect or contribution offered by the particle/matrix debonding, as seen here.

$$\bar{\varepsilon} = (1 - f)e^m + f\varepsilon^p + f\varepsilon^{\text{int}}$$ (35)

Where:

- \(\bar{\varepsilon}\) = Average Strain
- \(e^m\) = Strain in Matrix
- \(\varepsilon^p\) = Strain in Particle
- \(\varepsilon^{\text{int}}\) = Particle/Matrix Debonding

These standard equations can then be further advanced to include not only a matrix and single particle interaction, with the necessity for the particles to be of the similar dimensions (equation 36).

$$\sigma' = (1 - f)\sigma^m + \sum_N f_N \sigma^p_N$$ (36)

The calculation for the average strain is more involved and incorporates Compliance Tensors, shown in equation 37. These tensors can only be incorporated for the modelling when the particles and matrix are assumed or known to be linear elastic.

$$\varepsilon' = M^m: \sum_N f_N \{ (M^p_N - M^m): \sigma^p_N + \varepsilon^{\text{int}}_N \}$$ (37)

In which:

- \(M^M\) = Elastic Compliance tensor of the matrix
- \(\Sigma f_N\) = Total particle volume fraction
- \(M^p_N\) = Elastic Compliance tensor of type-N particle
- \(\sigma^p_N\) = Average stress of particle type-N
- \(\varepsilon^{\text{int}}_N\) = Uses approximations for evaluation (e.g. without any debonding \(\varepsilon^{\text{int}}_N = 0\))

This modelling process incorporates a variety of properties and gives a very detailed analysis of the stress and strain that are produced within a particle reinforced composite. However, for fibre reinforced RPC it is difficult to prove any associations between the Mori-Tanaka theory and what is known about the composition of RPC. The focus of the Mori-Tanaka on high volumetric fractions of composites, sometimes up to 60% of the
volume is far beyond the expectations of this project. The physical testing of this research is concentrating on RPC with a volumetric fraction of 5% fibre inclusions, up to 1/12 that used by Tan et al, 2005.

Research conducted into possible improvements to the Mori-Tanaka theory explored the use of fourth order orientation tensors (Benveniste, 1987), however, it still had similar areas of concern as discussed where the model focussed on composite materials, with high volumetric fraction. It is for these reasons that the Mori-Tanaka theory was not included into this report, the amount of time and resources necessary to alter the theory to suit RPC would not be of any benefit to the overall aim of the project, to incorporate several models and provide validation with physical test results.

It is now known which modelling techniques are applicable to RPC and those which can no longer be employed for the research of RPC within the scope of this project. This understanding of the models researched and analysed by this report, gives the relevant scope to allow for the develop a range of results from the modelling techniques. By developing these theoretical results it allows for the comparison to practical results giving validation of the modelling techniques.

IV. Comparison of Theoretical Modelling

Provided above is a detailed description of the various models that are available, also highlighted was the fact that only certain models were directly relevant to this research project. Below are the results and explanations of each that have been obtained from modelling techniques mentioned in this report.

A. Orientation Model

The analysis of the equations for the Orientation Model were carried out using MATLAB to calculate the mathematics, the code used for this report is contained in appendix A. The basic modelling techniques discussed earlier have been individually developed and the results for each compared in several graphs as detailed in this section. The first comparison made between the models was to look at the change in the equivalent Young’s Modulus, with a changing Young’s Modulus of the Matrix, as shown in Figure 10 - Comparison Graph for Basic Models. These tests were carried out with the following values; the values for the matrix were taken from the report by Dugat et al, 1996;

\[
V_m = 0.95 \ (95\%) \\
\nu_m = 0.23 \\
V_f = 0.05 \ (5\%) \\
E_f = 200 \ \text{GPa} \\
\nu_f = 0.28
\]

![Comparison of Basic Models](image)

**Figure 10 - Comparison Graph for Basic Models**
This graph gives a good understanding of the wide range of results that are available from the basic modelling techniques that have been employed in this report. The parallel model gives only a very small increase in the equivalent modulus while the series model assumes an equivalent modulus around 8GPa higher than the original matrix value, due to the inclusion of the steel fibres. During the validation of these results it will be determined which of the modelling techniques is more accurate when compared to the improvements gained in the physical testing.

Apart from altering the materials used as the matrix or fibre phases (discussed later in this report), the only other major change that can be made for the comparison of the basic models is to change the volumetric ratio of the fibres. For the physical application of RPC testing is not going past an RPC with a fibre ratio of only 5%, it is therefore redundant for any of the models to go into fibre content greater than 30%. Below is a table that details the modelling technique and the associated Young's Modulus obtained for the variation in the fibre content see Table 2 - Comparison of Basic Models for full details.

These calculations used the Young's Modulus of 31.9GPa from the results for HSC achieved by Price, 2009, and a Poisson's Ratio of 0.24 as used previously, all other values (except fibre ratio) have not changed. Also, for the calculation of the Hirsch equation the value of ‘x’ was taken to be 0.257.

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<td>0.26</td>
<td>0.27</td>
<td>0.28</td>
<td>0.29</td>
<td>0.30</td>
</tr>
<tr>
<td>Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Series</td>
<td>67.20</td>
<td>68.88</td>
<td>70.56</td>
<td>72.24</td>
<td>73.93</td>
<td>75.61</td>
<td>77.29</td>
<td>78.97</td>
<td>80.65</td>
<td>82.33</td>
</tr>
<tr>
<td>Parallel</td>
<td>38.74</td>
<td>39.14</td>
<td>39.54</td>
<td>39.96</td>
<td>40.39</td>
<td>40.82</td>
<td>41.26</td>
<td>41.72</td>
<td>42.18</td>
<td>42.66</td>
</tr>
<tr>
<td>Hirsch</td>
<td>46.05</td>
<td>46.78</td>
<td>47.52</td>
<td>48.26</td>
<td>49.01</td>
<td>49.76</td>
<td>50.52</td>
<td>51.29</td>
<td>52.07</td>
<td>52.85</td>
</tr>
<tr>
<td>Halpin-Tsai</td>
<td>61.47</td>
<td>62.94</td>
<td>64.42</td>
<td>65.90</td>
<td>67.39</td>
<td>68.89</td>
<td>70.39</td>
<td>71.90</td>
<td>73.41</td>
<td>74.93</td>
</tr>
</tbody>
</table>

As was expected the results for all the basic models gradually increase as the fibre ratio increases. This is due to the Young’s Modulus of the steel fibres being roughly 5 times larger than that of the concrete matrix. It is therefore common knowledge that if you place more of a material with a higher Young’s Modulus into a mix the higher the equivalent modulus should be. However, in practice this would prove a false assumption, for as the amount of fibres within the mix increased, up to and greater than 30-40% there would begin to be a limited amount of concrete matrix available. This matrix acts as a glue surround the fibres and holding the entire element together, if the amount of glue is reduced, but the number of elements that require securing increases then the element will easily break apart and crumble. A full list of results and calculations generated for the basic models, but not included in this report are in appendix B. With an understanding of the modelling processes it is now possible to validate these modelling processes with comparison to experimental results from Price, 2009.
V. Verification and Validation of Results

A. Comparison with Practical Results from Price, 2009

In parallel to this project another project was being conducted focussing on the physical testing of RPC, attempting to recreate the strengths and characteristics that have been generated by previous researchers. During the physical testing a list of batches to be developed and tested was generated that would give a range of result, involving various changes and improvements to the RPC mix (Table 3 - Test Batches (Price, 2009)).

<table>
<thead>
<tr>
<th>Batch No.</th>
<th>Description</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>RPC, with normal and fine sand aggregates</td>
<td>No pressure or temperature treatment</td>
</tr>
<tr>
<td>2</td>
<td>RPC, with normal and fine sand aggregates</td>
<td>No pressure or temperature treatment</td>
</tr>
<tr>
<td>3</td>
<td>HSC, no steel fibres included</td>
<td>No pressure or temperature treatment</td>
</tr>
<tr>
<td>4</td>
<td>HSC, no steel fibres included</td>
<td>No pressure or temperature treatment</td>
</tr>
<tr>
<td>5</td>
<td>RPC, with fine sand and fine quartz aggregates</td>
<td>No pressure or temperature treatment</td>
</tr>
<tr>
<td>6</td>
<td>RPC, with normal and fine sand aggregates</td>
<td>Post-set pressure added to specimens</td>
</tr>
<tr>
<td>7</td>
<td>RPC, with normal and fine sand aggregates</td>
<td>Post-set pressure added to specimens</td>
</tr>
<tr>
<td>8</td>
<td>RPC, with normal and fine sand aggregates</td>
<td>Post-set pressure added to specimens</td>
</tr>
</tbody>
</table>

This understanding of the test procedure to be followed in the lab gives a good understanding of what is expected of the modelling techniques and the alterations required. By making the same changes in the modelling as was made in the physical testing gives a good area for comparison, and allows for an accurate validation of the results to be undertaken. Table 4 – Experimental Results, Price 2009 shows the physical results (Price, 2009) that were achieved for the various experimental processes on RPC, while Table 5 - Theoretical Results, gives the outline of theoretical results obtained from the modelling process that were employed in this report.

<table>
<thead>
<tr>
<th>Test Specimen/Model</th>
<th>Young’s Modulus (E)</th>
<th>Poisson’s Ratio (ν)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Strength Concrete</td>
<td>31.9</td>
<td></td>
</tr>
<tr>
<td>RPC - Batch 1&amp;2</td>
<td>35.1</td>
<td></td>
</tr>
<tr>
<td>RPC - Batch 5</td>
<td>31.6**</td>
<td></td>
</tr>
<tr>
<td>RPC – Batch 7&amp;8</td>
<td>34.9</td>
<td></td>
</tr>
</tbody>
</table>

** - Indicates a poor test

<table>
<thead>
<tr>
<th>Model</th>
<th>Young’s Modulus (E)</th>
<th>Poisson’s Ratio (ν)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D Model</td>
<td>35.77</td>
<td>0.22</td>
</tr>
<tr>
<td>2D Model</td>
<td>36.65</td>
<td>0.24</td>
</tr>
<tr>
<td>Asymptotic Predictions</td>
<td>40.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Series Model</td>
<td>38.5</td>
<td></td>
</tr>
<tr>
<td>Parallel Model</td>
<td>33.3</td>
<td></td>
</tr>
<tr>
<td><strong>Hirsch Model</strong></td>
<td>35.1</td>
<td></td>
</tr>
<tr>
<td>Halpin-Tsai Model</td>
<td>38.5</td>
<td></td>
</tr>
</tbody>
</table>

As mentioned earlier (regarding the Hirsch Model) the original assumption for x was to be 0.4, therefore assuming around 40% of the fibres being orientated in series for composite materials, which was in fact reduced to around 26% (see equation 5), a 14% reduction. This original assumption of 0.4 is not correct, as expected, hence the development of a new value for x and the development of new values for the modelled equivalent Modulus. This reduction in the value of x, when used in the Hirsch Model, creates differences as shown in Table 6 - Alternate Values of x for Hirsch Model.
Table 6 - Alternate Values of 'x' for Hirsch Model

<table>
<thead>
<tr>
<th>Hirsch Model Calculations</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix</td>
<td>x=0.4</td>
<td>x=0.257</td>
</tr>
<tr>
<td></td>
<td>45.00</td>
<td>49.19</td>
</tr>
<tr>
<td></td>
<td>46.00</td>
<td>50.19</td>
</tr>
<tr>
<td></td>
<td>47.00</td>
<td>51.18</td>
</tr>
<tr>
<td></td>
<td>48.00</td>
<td>52.18</td>
</tr>
<tr>
<td></td>
<td>49.00</td>
<td>53.17</td>
</tr>
</tbody>
</table>

The reduction found in the value of x between composite and RPC materials, results in a reduction of around 0.7-0.9 in the equivalent Young’s Modulus. This reduction is minimal, and the assumption used for the original calculations (x=0.4), can be maintained for the purpose of this report. A better understanding of the small difference between the two values for x can be seen in Figure 11 - Comparison Graph for Hirsch Model. By observing the graph in figure, it shows that even with an increase in the matrix values, the equivalent value differs by a constant amount, which is roughly 0.9 as mentioned.

![Variation in x for Hirsch Model](image)

Figure 11 - Comparison Graph for Hirsch Model

However, for future calculations with RPC using the Hirsch Model, it is recommended that the altered value for x be used, or a new value developed before being the modelling techniques. As shown in equation 38, the three dimensional model develops another x value altogether.

\[
36.65 = x(31.9 \times 0.95 + 200 \times 0.05) + (1 - x) \times \frac{200 \times 31.9}{31.9 \times 0.05 + 200 \times 0.95}
\]

Yielding a value of x as;

\[
x = 0.48
\]

Care is therefore needed when using the Hirsch Model, as an increase in the equivalent Young’s Modulus (as from the practical results to the three dimensional model used here) can obtain results that vary. It is still difficult to know exactly what values to use for x, the Hirsch Model needs a greater amount of development against experiment results to fully understand and compare the orientation of the fibres with in an RPC matrix. It cannot be compared against the practical results obtained here; as they were the results used to develop the
initial value of \( x \) and so would yield exact solutions, giving an inconclusive solution. However, it can still be compared to the other modelling techniques the results obtained with these as has been discussed in this report.

Another area for comparison that can be exploited for all the modelling processes is the comparison between the changes that occur due to a change in the fibre ratio within the mix, similar to the results achieved in Table 2 - Comparison of Basic Models. However, this time it is represented by the graph in Figure 12 - Changes in Fibre Ratio and incorporates all modelling techniques. The matrix incorporates a value of 31.9GPa for the Young’s Modulus, while the vertical line represents the fibre ratio used in experimental testing. Here, the physical results (Price, 2009) are the horizontal lines and give various comparisons to the theoretical data.

![Comparison for changes in fibre ratio](image)

**Figure 12 - Changes in Fibre Ratio**

The black box surrounding the intersection of the vertical and horizontal lines, shows the meeting point for the experimental results. As expected the Hirsch Model travels directly through this intersection, as it used the experimental data to establish the variable \( x \). The two and three dimensional models established from the orientation model gives a close estimate to the experimental results, where the two dimensional model only overestimates the equivalent modulus by around one and the three dimensional model around a half. While the parallel model gives an underestimation of the final value. The Halpin-Tsai and Series models both give larger overestimations. This overestimation is to be expected with theoretical results and the range of models employed in this report give a larger margin in which these variations can augment. The tables of results used to develop the graph in figure, as detailed in appendix D.

**B. Comparison with Research Articles on Reactive Powder Concrete**

The intersection of the horizontal blue line and the vertical purple line show the meeting point which indicates the unfibred result (purple vertical line) and the new modulus (horizontal blue line) achieved from the addition of 3% steel fibres. Here the comparison to a report produced by Bonneau et al, 1997 whom again determined the variations that are possible due to the additions of steel fibre reinforcement in RPC. Figure 13 - Comparison to Bonneau shows the theoretical results from this report and how they corresponded to the physical results achieved by Bonneau et al.
This graph in Figure 13 - Comparison to Bonneau shows the Hirsch Model and with the two and three dimensional models also being very accurate in comparison to these results. While the parallel model is also relatively close to the equivalent modulus. The series and Halpin-Tsai models are both overestimating the equivalent modulus. The tables and results that were used for the development of the various graphs used in this section for the comparison of the modelling processes are located in appendix C.

C. Using Properties for Further Analysis

As mentioned earlier in this report it is possible once the material properties are known to develop a finite element analysis package that looks at the stress, strain and deformation of a specimen or structural member. This report looked at two possibilities with using RPC as the structural material. The first was an RPC beam, of the same dimensions as those used in the flexural tests in the lab, see Figure 14 - Beam Test Specimen in Ansys.

For ease of analysis the beam has been drawn as a two dimensional object, however, within Ansys the thickness of the specimen is incorporated so that it still performs as a solid, three dimensional beam. Note how
in the drawing of the specimen, no fibres have been included, this is due to the establishing of an equivalent Young’s Modulus and Poisson’s Ratio. For both of the Ansys examples the results achieved by the two dimensional Orientation Model have been employed, that is a Young’s Modulus of $E = 41.5 \text{ GPa}$ and a Poisson’s Ratio of $\nu = 0.24$.

As shown in Figure 14 the single face of the beam has been meshed, the contact points from the base of the test machine have been set to offer no displacement in the $y$-direction, while the load is applied to the top-centre of the beam in the negative $y$-direction. The specimen has also been restricted from rotating about the $x$-axis (into the page), as this would be nearly impossible when the beam is sitting flat on the apparatus. With the specimen ready for analysis it is now possible to develop some results regarding the performance of the beam.

![Figure 15 - Von Mises Plot for Test Beam used by Price](image)

The plot shown here in Figure 15 - Von Mises Plot for Test Beam used by Price is what Ansys calls a ‘Von Mises Plot’ which gives a contour plot of the stress as it applies to the test specimen. In this case it is for an RPC beam that is being tested in tension. The two points of increased stress on the bottom level of the beam indicate the rollers which support the beam, while the load is applied to the upper level of the beam. The lighter blue indicates that these three points on the beam are under higher levels of stress, while the other points along the beam indicate little to no impact due to the testing. This indicates that the centre of the beam is under most stress and so should be the failure point during physical testing, this will be confirmed later in the report during the verification and validation of the results.

By recalling the Ansys tests conducted on an RPC beam (see Figure 15), it was estimated that the centre point of the beam would be the location of failure during practical testing. Figure 16 - Failed Beam (Price, 2009) indicates a physical RPC beam that has been tested in tension, with an identical set up to that demonstrated in the Ansys function (two support points and a single point of load at the top).

![Figure 16 - Failed Beam (Price, 2009)](image)

As expected the beam fails along the centre line, where the major points of stress were identified by the Ansys model developed earlier. The crack is not straight from top to bottom, and appears to stop before reaching the top of the beam (the beam in this image is upside down, to how it was placed for testing). It can be assumed that this occurred due to the presence of the steel fibres within the mix, which acted as binders and did
not fail when the concrete began to fracture. The tests conducted in the experimental analysis of these beams only included a maximum downward force to establish the Modulus of Rupture, it was originally planned to incorporate the deflections as well to provide a more detailed comparison to the Ansys model established in this report. However, due to constraints regarding the physical testing the apparatus and resources for the calculation for deflection was not available; therefore further comparisons cannot be made to this report. However, this area is a good area for further development in the analysis of RPC, as discussed later in this report.

The scope of future work may aim to include the determination of strength with the modelling process, but were not developed in this report. Bearing this in mind it was the purpose of this Ansys modelling is to give better understanding to the possibilities that arise from the modelling process employed within this project; even though the modelling processes do not fully develop the RPC properties into a fully developed analysis with strength characteristics to accompany the material properties. The material properties of RPC can still prove to be extremely useful if they are employed in the correct way. A finite element analysis can still be conducted by employing the equivalent properties into a program such as Ansys. Improvement of the Fibre Reinforced RPC Mix

D. Alternate Fibres

This project has conducted a brief analysis into the change of material properties that is caused by the use of alternate fibres to steel fibres. The two types of fibres are glass and natural fibres. Also, the new fibre materials bring with them other areas of concern, but also improvements to RPC as discussed below.

1. Glass Fibres

For the calculation of the material properties of RPC with glass fibre reinforcement in place of steel fibres, the glass fibres are assumed to have a Young’s Modulus of \( E = 78.5 \text{GPa} \). While within the report the glass fibres were mentioned to be 50mm in length and have a diameter of 13.8µm. Meaning that they are long slender fibres. Being that these fibres are long and slender, problems may be faced when conducting physical tests on incorporating these fibres within RPC. Figure 17 - Example Glass Fibres, shows an example of the types of fibres that may be used in the physical testing of RPC with glass fibre reinforcement. If the glass is to undergo the same rigorous mixing process as the steel, they may easily crush, leaving fibres within the mix that have smaller dimensions than is assumed, and the mechanical properties of the material may also be reduced as a result of the mixing process. However, for the comparisons within this report it is assumed they are of the length and properties discussed, and mix thoroughly through the RPC matrix.

2. Natural Fibres

Another possibility for fibre substitution is the use of fibres made from a polymer or natural product. This report takes the natural fibre, Jute, and uses the material properties of this fibre. Jute is a naturally occurring vegetable fibre that uses several strands woven together, forming a single strong strand (Wambua et al, 2003), the fibres are assumed to take on the form shown in figure, with the fibres having the same length as for the glass fibres. While the Young’s Modulus is taken to be \( E = 30 \text{GPa} \). An example of the form of Jute fibres is shown in figure 18, where a mesh created of these fibres is shown.

Figure 17 - Example Glass Fibres
The benefit of these fibres, is that they are naturally occurring and are extremely cheap to produce. This provides a product that is environmentally friendly, while also not becoming economically unfriendly. With an understanding of the types of fibres to be used in this comparison and some necessary data on each it is now possible to develop a comparison between the three fibre types; steel, glass and jute, to understand how each performs as a reinforcement fibre for RPC.

**E. Explanation of Results from Alternate Fibres**

The use of alternate fibres achieves the results, as shown here in Table 7 - Equivalent Modulus from Alternate Fibres, the modelling techniques employed for this comparison included the series, Hirsch, two dimensional and three dimensional models to provide a range of results for the use of alternate fibres. The Hirsch model will use a value for \( x=0.257 \), as was previously established from the practical results on steel fibre reinforced RPC. All tests used an original matrix value of \( E=31.9\text{GPa} \), while the fibre ratio was taken to be 5%.

<table>
<thead>
<tr>
<th>Modelling Technique</th>
<th>E (GPa) (Steel Fibres)</th>
<th>E (GPa) (Glass Fibres)</th>
<th>E (GPa) (Jute Fibres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Series Model</td>
<td>38.50</td>
<td>34.23</td>
<td>31.81</td>
</tr>
<tr>
<td>Hirsch Model</td>
<td>35.10</td>
<td>33.22</td>
<td>31.80</td>
</tr>
<tr>
<td>Two Dimensional Model</td>
<td>36.65</td>
<td>33.85</td>
<td>31.76</td>
</tr>
<tr>
<td>Three Dimensional Model</td>
<td>35.77</td>
<td>33.79</td>
<td>31.73</td>
</tr>
</tbody>
</table>

As shown in Table 7, the reduction in the Young’s Modulus of the Fibre, leads to a significant reduction in the equivalent modulus of the concrete and fibre mix. The Jute fibres give nearly no improvements to the mix, not even giving 2GPa for the inclusions, while the glass fibres still give a reduction but not quite as severe as the natural fibres. This was to be expected in the results, and future use of these fibres needs to be weighed against the overall factors that are incorporated with their use. Further analysis should include the interaction between the fibre and concrete, while the costs and availability will also play significant roles in the future employment of these fibres.

**VI. Summary and Conclusions**

**A. Work Completed**

This report has developed a range of modelling techniques that have been employed in an attempt to analysis the new age construction material RPC. Initially this report focussed on the research and previous analysis that has been conducted on both RPC and the modelling processes that are possibly available. From this research it was possible to establish which techniques would be of the most use when analysing the fibre reinforcement of RPC. With an understanding of the possible techniques available, it was now necessary to establish a detailed understanding of the processes involved. This involved detailed research into previous uses of the technique, as well as the development of any further improvement or assumptions that could be made to give a better representation of the behaviour or RPC. With an understanding of the modelling techniques and how they can
be implemented it is now possible to establish various results from these models. Initially the modelling processes used to establish the theoretical analysis are compared just to themselves, from which a better understanding of the accuracy of each model is gained. It also allows for the development of a margin in which it is hoped that the experimental results will fall. With the models known and a basic analysis conducted, it is now necessary to provide validation of the results with verification against experimental data.

This comparison to the experimental results obtained has lead to the development of various questions in the modelling processes that can be analysed to better understand the intransient nature of RPC and the fibre reinforcement that is directly related. These areas of questions provide possibilities for further research work in the area of modelling of RPC, as is discussed below.

B. Recommendations for Future Work

This report has only begun to scratch the surface of modelling for RPC, and there are several areas (as indicated through the report) that require larger amounts of analysis and research to be fully understood. A major area is in the use of more accurate testing processes such as the Mori-Tanaka and Nonlocal Theory of Elasticity. These modelling techniques may be useful for the use when analysing RPC, however the nonlocal theory will require a large amount of mathematical operations and manipulations to fully develop a constitutive model for RPC. However, Mori-Tanaka will require investigation into the use of this technique on a fibre-matrix mix that uses lower amounts of fibre inclusions than previous research articles have.

This report also conducted some research on the possibility of using fibres other than steel for the reinforcement of RPC. Though it was possible to obtain a comparison to the results for steel fibres, further research on the physical testing of these fibres would prove more appropriate. There may be further problems encountered in the practical uses that cannot be determined with just theoretical modelling, such as the breaking up of glass fibres in the mixing or unacceptable lumping or degradation of the natural fibres.

Also, a major discussion point of this report was the orientation of the fibres and how this can be instilled within the modelling process. It would be beneficial to conduct physical testing on the fibre orientation to give a better understanding of how well the theoretical results compare to actual results. By producing RPC specimens the pieces could have x-ray scanning to give imagery of the fibres within the concrete and how they are orientated. This would be beneficial, especially for the Hirsch Model which focuses on the fibres orientated in either series of parallel.

This report also employed brief comparison to the development of Ansys models to the testing from Price, using the rectangle beam sections. Further development in this area could use the maximum load imposed on the beam, achieving a relevant deflection as a result of this load. This would allow the modelling processes to be directly related to the strength parameters achieved in a finite element model.

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**Appendices**

A. MATLAB code for the development of Orientation Model Results
B. Table of results for Basic Model Calculations
C. Fibre ratio comparison tables