Observations of Supersonic Mixing Layers

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Large-scale structures in turbulent mixing layers were first discovered in subsonic flow. Later research proved the existence of similar structures in a supersonic wake, with interest driven by the effect of these structures on supersonic mixing and the applications of improved mixing to developing technologies. Again led by subsonic flow studies, structures exhibiting a characteristic angle of inclination were also found to convect into the supersonic wake. A symmetric supersonic flow over a horizontal splitter plate has been studied by a number of past thesis students at UNSW@ADFA, but the nature of an asymmetric flow is less well understood. This thesis will attempt to validate the results of a Mach 2/Mach 3 asymmetric flow study by Janicke (2002). Pressure fluctuations will be measured to attempt to identify a shedding frequency of large-scale vortices from the splitter plate trailing edge. A Pitot tube pressure survey will be conducted to obtain velocity profiles for the wake at different downstream positions. A new schlieren visualisation setup with a high speed camera will be used to achieve time-evolution images of the flow.

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Nomenclature

\[ Re = \text{Reynolds number} \]
\[ \rho = \text{fluid density, kg.m}^{-1} \]
\[ V = \text{freestream velocity, m.s}^{-1} \]
\[ l = \text{characteristic length, m} \]
\[ \mu = \text{dynamic viscosity, s.kg}^{-1}.m^{-1} \]
\[ R = \text{specific gas constant, J.kg}^{-1}.K^{-1}; \text{assumed to 287.06 J.kg}^{-1}.K^{-1} \text{ for air} \]
\[ M = \text{Mach number} \]
\[ M_t = \text{pre-shock Mach number} \]
\[ U = \text{freestream velocity, m.s}^{-1} \]
\[ a = \text{local speed of sound, m.s}^{-1} \]
\[ M_c = \text{convective Mach number} \]
\[ U_c = \text{convective velocity, m.s}^{-1} \]
\[ Re = \text{unit Reynolds number} \]
\[ T_0 = \text{stagnation temperature, K} \]
\[ p_0 = \text{stagnation chamber pressure, Pa} \]
\[ p_1 = \text{free stream static pressure, Pa} \]
\[ p_{02} = \text{Pitot probe pressure, Pa} \]
\[ \gamma = \text{ratio of specific heats; assumed to be 1.4 for air} \]

\section{Introduction}

Research into atmospheric propulsion continues to explore and refine the supersonic combustion ram-jet (scramjet) engine as the future of high-speed, high-altitude travel. In the past half-century, however, progress in this field has been restrained by the lack of understanding in related fields of study. One of these fields includes the mixing interactions of supersonic streams.

\subsection{A. Importance of Scramjet Engines}

The scientific community has been interested in an advance on the conventional jet turbine since the end of the Second World War. An intermediate concept, the ram-jet, uses a ram effect to compress the intake air prior to a subsonic combustion process. This subsonic stage requires the intake air to be slowed down significantly in an inefficient, high-drag process. The advantage of a scramjet is that the combustion occurs at supersonic speed, and their efficiency improves with higher speeds (Anderson 2007).

In current experimental tests, successful scramjets have only achieved a small value of net thrust. Despite this limitation, scramjets are still the subject of interest because of the advantages they have over conventional technologies such as rockets. As scramjets use intake oxygen, even at high altitudes, they do not need to carry their own supply in addition to other fuel, significantly increasing the weight and space available for payload. The usefulness of scramjet technology is represented in Fig 1.

\subsection{B. Relevance of Supersonic Mixing Layers}

With the potential for a fuel efficient, high-altitude, hypersonic engine recognised, research into supersonic interactions is necessary to advance our understanding of what occurs inside the engine. Due to the supersonic flow speed, the residence time of fuel in a standard size combustor is in the order of $10^{-3}$ to $10^{-4}$ seconds. To increase the efficiency of the combustion, without the drag penalty of a bigger device, the mixing of the fuel and airflow is the focus of investigation. Although there is much information available about the mixing interactions of subsonic flows, the same knowledge does not exist for supersonic flows (Gutmar, Schadow et al. 1995).

\subsection{C. Common Parameters}

It is useful to define several parameters that describe aspects of previous work and will be used in this thesis. The first of these is Reynolds number, a ratio of inertial forces to viscous forces, that can indicate whether a flow has remained laminar or transitioned to turbulent. The parameter takes the form of (Anderson 2007):

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Comparison of scramjets and other technologies, where $I_{sp}$ represents the specific impulse.}
\end{figure}
Another valuable parameter, especially used in describing supersonic flows, is Mach number. It is the ratio of the local freestream velocity to the local speed of sound, given by (Munson, Young et al. 2006):

\[ M = \frac{U}{a} \]

II. Review of Previous Work

A. Introduction

The particular area of interest for this thesis is the mixing behaviour of a supersonic flow downstream of a finite trailing edge. Two supersonic flows are separated by a splitter plate, inside a wind tunnel, and rejoin in the wake of this plate. Previous theses by students at UNSW@ADFA were concerned with a symmetric supersonic flow, where the flow over the top and bottom of the splitter plate is the same speed (Perry 1999; Hughes 2000). In both these studies the separated flow was travelling at Mach 2. Figure 2 shows a basic graphical representation of the flow characteristics. The two diagonal lines extending from the axes origin represent the recompression shockwaves generated when the flow rejoins behind a finite edge.

This work was extended to study an asymmetric flow, where the two co-flowing streams travelled at Mach 2 and Mach 3 (Janicke 2002). The asymmetric flow presented a fresh set of characteristics due to the pressure difference between the two different flow speeds. The Mach 2 of the wake was characterised by an expansion fan and recompression shock; whereas the Mach 3 flow transited through a single oblique shock shockwave. The asymmetric flow also produced a deflection of the wake towards the Mach 3 side of the flow, shown in Fig. 3.

Another parameter has been defined to describe the characteristics of the wake. The convective Mach number characterises the compressibility effects in the mixing layer between flows of different speeds and describes the convection velocity of organised motions in the wake (Papamoschou and Roshko 1988; Smits and Dussauge 2006). It is a value that represents the flow as a whole and does not vary with downstream position. It is expressed below:

\[ M_c = \frac{U_1 - U_c}{a_1} \]

Where:

\[ U_c = \frac{a_1 U_2 + a_2 U_1}{a_1 + a_2} \]

\[ U_1, U_2 = \text{freestream flow velocities above and below the mixing layer} \]

\[ a_1, a_2 = \text{local speed of sound above and below the mixing layer} \]
The theses briefly described above have all focused on investigating the nature and effects of large-scale structures in the supersonic mixing layer. The following sections introduce these phenomena.

B. Vortex Structures

Brown and Roshko (1974) discovered that, for all density ratios between two subsonic plane mixing gases, the mixing layer was dominated by large, periodic structures. Furthermore, these structures convect at almost constant speed and increase in size, but the spacing downstream increases discontinuously, suggesting the amalgamation of these structures. These same structures were subsequently shown to exist in a supersonic turbulent wake (Bonnet and Chaput 1986). Figure 4 clearly shows the large-scale vortex structures discovered by Brown and Roshko (1974) in a subsonic wake.

Note that the vortex structures in Fig. 4 are visible immediately following the trailing edge. Bonnet and Chaput (1986) suggested that large-scale structures in supersonic wakes do not develop until far downstream but further research proposed that the compressibility effects of supersonic flow caused the suppression of vortex structures. It was demonstrated that supersonic mixing layers with low convective Mach numbers behaved with incompressible characteristics, exhibiting the two-dimensional, large-scale vortex structures observed by Brown and Roshko (1974). At high convective Mach numbers, however, the mixing layer is highly three-dimensional with little large-scale organization apparent, a change due to compressibility and not related to the Reynolds number (Clemens and Mungal 1992; Clemens, Petullo et al. 1996).

The wake behind a flat plate in both subsonic and supersonic flow regimes was studied by Althaus (1990) when attempting to ascertain the conditions under which a vortical structure would form. The study varied the surface roughness of the splitter plate to determine the effect on the wake flow. Figure 5 shows the distinctive Karman vortex sheet shed either side of the wake centreline under the Althaus’ ideal conditions. Contrary to the observations of the downstream mixing layer by Bonnet and Chaput (1986), Althaus (1990) did not observe a vortex sheet for a smooth surface splitter plate. It has been suggested that vortex structures do indeed exist for in the wake of a smooth splitter plate, but do not manifest themselves until further downstream (Hughes 2000).
Visualisation conducted by Perry (1999) and Hughes (2000) of the symmetric wake detected vortical structures similar in appearance to the Karman vortex sheet, shown in Fig. 6. These vortices did appear to alternate either side of the centreline as they were shed and the distance between the vortices increased with downstream distance.

The examination of an asymmetric wake by Janicke (2002) appeared to display an inequality in the vortices shed on each side of the centreline. Although visualisation showed vortices generated on the Mach 2 side of the flow, the presence of vortices on the Mach 3 side could not be verified (Janicke 2002). The asymmetric case is shown in Fig. 7.

C. Inclined Structures

Again extending subsonic flow research, structures in the turbulent boundary layer that, at high Reynolds number, resemble hairpin vortices angled with the flow direction were found by flow visualisation (Head and Bandyopadhyay 1981). Further work in the supersonic field found that similar structures existed in the supersonic turbulent boundary layer, extending from the plate surface to the layer ceiling at a consistent inclination of 45°–60° (Spina, Donovan et al. 1991). These inclined structures have been attributed to a high Reynolds number horseshoe vortex. Mathematical and experimental evidence demonstrated the existence of a vortex tube, Fig. 8, that attaches both ends to a solid boundary, such that a loop is formed (Theodorsen 1955).

Theodorsen explained the shaped of these structures by their exposure the lift and drag forces exerted by the freestream flow. Such forces are resisted because the vortex has a low pressure core and this, combined with the attachment of both ends to a surface, results in a downstream inclination.

Investigation of a symmetric supersonic wake revealed that these inclined structures also exist in the mixing layer (Perry 1999).

D. Effect of Large-scale Structures on Mixing

The low growth rates of supersonic mixing layers presents a real problem to supersonic combustion, but the presence of large-scale structures has been found to have a positive impact on mixing (Gutmar, Schadow et al. 1995). Brown and Roshko (1974) described the ‘entanglement’ of non-turbulent fluid into the turbulent mixing layer during the formation of large, coherent eddies. Also, the smaller instabilities imbedded in the large-scale structure did not enhance the ‘ingestion’ of non-turbulent fluid but did contribute to its ‘digestion’. Indeed it is the small scale instabilities that accomplish the fine-scale, molecular mixing (Dimotakis and Brown 1976).
Compressibility effects elongate the large-scale structures and suppress their evolution in the near-trailing edge wake, meaning that the mixing layer is less effective at ingesting non-turbulent fluid. As a result, the mixing layer growth rate is reduced, observed by Clemens and Mungal (1992). Increasing Reynolds number appears to reduce the strength of vortices shed from the trailing edge and elongate inclined structures such as they are less effective at engulfing freestream fluid (Perry 1999; Janicke 2002). To counter the reduced mixing layer growth rate caused by compressibility, geometric changes can be made to the splitter plate to encourage increased mixing. Such geometric modifications may include a non-plain trailing edge or the addition of vortex generators (Dolling, Fournier et al. 1992). The use of a splitter plate with trailing edge vortex generators was found to have a positive effect on the growth rate of a symmetric mixing layer (Perry 1999).

E. Project Aim

Previous theses by Perry (1999) and Hughes (2000) have studied the wake flow characteristics of a symmetric supersonic flow over a splitter plate. More recently a Mach 2/Mach 3 asymmetric flow was examined by Janicke (2002). Since these studies a new, one million frames-per-second Shimadzu camera has become available for use in schlieren visualisation of the flow.

This thesis will examine the mixing layer of a Mach 2/Mach 3 asymmetric flow over a splitter plate. Data are to be collected and analysed on the:

1. Evolution and convection of inclined structures from the near-trailing edge boundary layer to the shear layer.
2. Shedding of large scale vortex structures in the near wake.
3. Effect of trailing edge geometry on the above structures and wake growth.

The findings of this thesis will be used to validate those of Janicke (2002) or suggest alternative conclusions where appropriate.

III. Experimental Equipment

As a continuation of previous work conducted in the School of Aerospace and Mechanical Engineering by past honours students. The Supersonic Wind Tunnel (SSWT), located on campus, will be used to generate an asymmetric supersonic flow over a selected splitter plate at different Reynolds numbers. The equipment placed into the wind tunnel consisted of an upper and lower liner, to generate the required flow speeds, and a horizontally mounted splitter plate to divide the two flow regions for a distance.

A. Supersonic Wind Tunnel

The SSWT, shown below in Fig. 9, consists of a compressor plant, high pressure reservoir, control valve and circuit, stagnation chamber, first throat, test section, second throat, subsonic diffuser and muffler (Janicke 2002).

Three high pressure reservoirs are charged to an operating pressure of 1500 kPa by two oil-free compressors. These reservoirs supply pressure of 100 kPa to 500 kPa to the stagnation chamber. The stagnation pressure referred to in thesis study applies to the gauge pressure in the stagnation chamber. Since the last study in 2002, the SSWT has been fitted with a proportional-integral-derivative (PID) controller that replaces valve control system used for previous studies.
B. Tunnel Test Section

The air from the stagnation chamber is accelerated by passing through the converging-diverging nozzle established by a pair of removable liners. The test section, shown in Fig. 10, extends from the first throat to second throat and has a cross-sectional area measuring 155 mm x 90 mm. A circular viewing window measuring 148 mm in diameter located towards the downstream end of the test section. On one side of the test section, two ports are located in the test section wall in order to allow static pressure measurements to be conducted on the upper and lower sides of a horizontal splitter plate.

The ceiling of the test section accommodates three entry points that will be used to insert a Pitot probe apparatus. These entry points are located at 83.5 mm, 133.5 mm and 183.5 mm behind the trailing edge of the splitter plate.

C. Achieving Asymmetric Flow

To achieve the desired flow speed, aluminium liners designed by Associate Professor Sudhir Gai are inserted in the first throat of the SSWT. Opposing the leading end of the splitter plate, these liners vary the cross-sectional area of the throat experienced by the flow to achieve the Mach number intended – Mach 3 on the bottom and Mach 2 on the top. Due to the existence of a boundary layer, the effective cross sectional area is actually slightly smaller and results in a variation in the Mach number actually achieved. The Mach 3 flow is established on the bottom so that the mixing layer is deflected downwards and reduces interference with the Pitot pressure probe used to measure pressure fluctuations (Janicke 2002). Figure 10 shows a plot of the liner shapes.

Figure 10. Test section dimensions (Janicke 2002).

Figure 11. Shape of test section liners (Janicke 2002).

D. Splitter Plate Models

Two models are available to observe the effect of the splitter plate’s trailing edge geometry on organised structures in the mixing layer. The plain trailing edge model and a saw-tooth trailing edge model are shown below in Fig. 13 and Fig. 14 respectively. Both of these geometries were used in the previous study by Janicke (2002). Both splitter plate models are 480 mm long, 90 mm wide and have a maximum thickness of 12 mm. The trailing edges have a thickness of 1 mm. The length of these splitter plates extends them upstream of the first
throat and into the reservoir, dividing the Mach liners installed in the throat. The longitudinal cross-section of the models resembles a symmetric airfoil.

Since the flow is asymmetric, the pressures on each side of the splitter plate are unequal. The splitter plates are supported by three pins on both sides of the test section, with the last pins located about halfway down the splitter plate, 244 mm from the trailing edge. The lack of support to the trailing edge and the pressure difference of the asymmetric flow generated oscillations of the trailing edge that were observed by Janicke in 2002. The solution was the design and installation of two slotted plugs, one on each side of the plate, that sit in a wall recess closer to the trailing edge. These plugs do generate weak shocks but are necessary for the observation of an asymmetric flow free of model oscillations (Janicke 2002).

During a familiarisation with the SSWT it was found that the slotted plugs for the splitter plates were missing and would have to be remade. An inspection of the approximate location for these plugs on the splitter plate showed a plate thickness of about 2 mm. Michael Jones, a School technician, stated that the School workshop now owned an electric discharge machine which could be used to drill a 1 mm diameter hole into the side of the plate. This would only leave about 0.5 mm either side of an inserted pin if completed. Such a measure was not available in 2002.

Although removing the intrusions of the slotted plugs was an obvious advantage, there is also considerable risk. Drilling the 1 mm holes in the right location on the splitter plate to line up exactly with pins to be fitted to the SSWT wall would waste valuable time if only slightly inaccurate. Also, if the splitter plate did not withstand the stress caused by the pressure difference at 400 kPa, or lower, a new splitter plate would need to have been manufactured. It was decided to proceed with the re-manufacturing of the slotted plugs given that: the previous study used a slotted plug setup; a new method is not required to validate the 2002 work; and, the consequences of a damaged splitter plate would cause significant delay. The fit of the new plugs to the plain edge splitter plate is shown in Fig. 15.
IV. Experimental Methods

A. Schlieren Visualisation

1. Technique

Schlieren visualisation is a non-intrusive technique that uses the refraction of light to identify density gradients in a flow (Holder and North 1963). The test section of the SSWT is bounded on either side by two circular windows which provide a view of the splitter section trailing edge and the downstream flow. The supersonic flow passing through the test section contains density gradients that exist across the shockwaves, expansion fans and the large-scale structures present. The visualisation method entails passing a collimated beam of light through the test section, where refraction of the light occurs across the density gradients. On the other side of the test section the light beam is focused onto an image plane with a convergent mirror. Unrefracted light will pass exactly through the focal point of the image plane, whereas any refracted light will not. This refracted light can then be obstructed by a knife-edge at the focal point. A diagram of an example arrangement is in Fig. 16 demonstrates this concept.

![Schlieren Arrangement Diagram](image)

The light beam then proceeds to the camera, less the obstructed light which will appear as a dark region on the image. The light that was refracted in the unobstructed direction will augment the unrefracted light and appear as light regions on the image. The density profile of the flow is indicated by the contrast between the light and dark regions on the resulting image.

The direction from which the knife edge is imposed can be used to vary the direction of density gradients that are visualised. For example, a vertical knife-edge is sensitive to density gradients in the horizontal direction. The vertically inclined knife-edge setup will be used to visualise large-scale structures relative to their local axis in the wake (Holder and North 1963; Hughes 2000; Janicke 2002). Much of the visualisation was conducted with a ‘circular’ knife edge. A ‘circular’ knife edge cut-off is sensitive to the refraction of light in all directions, providing information on density gradients in all directions in the flow.

The use of a high-speed camera is a new addition to this study, previous work having been completed with Polaroid film. The new camera is a Shimadzu Hypervision HPV-1. The shutter speed of the one million frames-per-second camera, and its ability to take successive images without the replacement of Polaroid film, means that we can achieve multiple frames of the flow and provide greater insight into the evolution of organised motions. Each run of the camera generated 102 images, taken between 250,000–500,000 frames per second, providing a reduced-speed video representing approximately 200–400 μs in the flow. Examples of circular and inclined knife edge images are shown in Fig. 17 and Fig. 18; the trailing edge of the splitter plate imposes from the left-hand side.

![Circular Knife Edge Schlieren Image](image)

![Inclined Knife Edge Schlieren Image](image)
2. Complications in analysis

Analysis of the schlieren imagery was a qualitative process, affected by the resolution of the images. Although the camera has a very high frame rate, it is because of this capacity that the resolution of the images is far below that of a commercial digital camera.

B. Pressure Fluctuation Measurements

This study attempted to identify the shedding frequency of the vortices from the trailing edge by conducting a frequency analysis on downstream pressure measurements for all three stagnation pressures. The experimental procedure for this is taken from previous work by Perry (1999), Hughes (2000) and Janicke (2002). The purpose of taking these measurements is to identify any dominant frequencies that exist for the organised structures in the mixing layer (Gai, Hughes et al. 2002).

1. Equipment and method

A Pitot probe containing the Kulite XCQ-062 100VG (100 psi) transducer was mounted in a bracket that inserted into the ceiling of the wind tunnel (Fig. 19). The probe was positioned in the vicinity of the expected vortices and a two second signal sample was recorded flows of varying stagnation pressure. The exposed end of the Pitot probe is shown in Fig. 20. The transducer was sampled at a rate of 1 MHz.

The vertical position of the probe was confirmed from schlieren images taken. Although the previously mentioned slotted plugs did prevent the trailing edge from visibly oscillating, the pressure differential did deflect the trailing edge downwards to a stable position. Since the only reference point in the schlieren imagery is the trailing edge of splitter plate, it is difficult to accurately position the probe in the flow based on measurements taken from a schlieren image. A more reliable and less time consuming method is to confirm the probe’s position with a schlieren photograph and use the signal that is recorded in that position.

2. Analysis

The pressure signal detected by the transducer was analysed with a discrete Fourier transform (DFT). This is computed by using a fast Fourier transform (FFT) algorithm in MATLAB to transform the time domain representation of the signal into a frequency domain representation. The FFT algorithm decomposes the sample signal into its constituent frequencies and plots the transformed amplitude of these frequencies. Such a computation can identify the fundamental frequencies in the signal (Hanselman and Littlefield 2005).

C. Velocity Profiles

1. Equipment and method

A traversing Pitot probe survey was conducted vertically through the flow at different downstream positions at a stagnation pressure of 300 kPa. The purpose of this pressure survey was to determine the pressure in the flow, from which the velocity can be calculated and collected into a profile across the vertical range.

There are three entry points in the ceiling of the test section, shown previously in Fig. 10, through which the Pitot probe can be inserted. Furthermore, numerous probe extensions provide additional flexibility in the downstream position of each survey. The surveys were conducted at the downstream positions of 4.5 mm, 11.5 mm, 22.5 mm, 50 mm, and 76 mm. This setup was very similar to Janicke (2002) and is shown in Fig. 21.
The Pitot probe was mounted in a test section bracket that allowed free movement of the probe (Fig. 22). The top end of the probe was also secured to a stepper motor apparatus mounted on top of the test section (Fig. 23). The stepper motor was controlled by a Labview program, written by the school’s technical support section. A series of dry run experiments were conducted to determine: the average distance travelled by the probe over 1000 motor ‘steps’; and the number of ‘steps’ in a 30 mm traverse. The results from these trials were consistent with each other used to operate the stepper motor. The pressure samples were taken at 31 vertical locations over a range of approximately 30 mm. It required six runs of the wind tunnel to complete a full vertical traverse.

2. Analysis
This high speed data recorded at each vertical position by the transducer were stored in separate data file created by the ACQ program. These files were then loaded into MATLAB and, using the code in Appendix D, saved into a single .MAT file that contained all the information for a profile at a given downstream position. The data were converted to a pressure value, using the calibration information contained in Appendix E, and averaged to find the mean pressure recorded at each position.

The minimum pressure was assumed to represent the wake centreline. The pressures were converted to velocities using the freestream conditions on their respective sides of the wake and realigned in plot such that the minimum velocity defect corresponded to the wake centreline.

3. Error
As the wake is deflected downwards by 11°, the Pitot probe is not perpendicular to the flow. This will result in a minor error because the Pitot probe equations used in Section V assume a normal shockwave forming in front of the probe. The error in the velocity measurements could be up to 5%.

V. Validation of Test Section Flow
The 2002 study by Janicke validated the then-new Mach 3 liner for use in the wind tunnel. His thesis demonstrated that a laminar flow was produced by the new liner and confirmed that a turbulent boundary layer was generated at the trailing edge of the splitter plate for both liners.

Although the operating characteristics of the wind tunnel have not changed, it is necessary to validate the Mach numbers generated on either side of the splitter plate. The stagnation pressure of the wind tunnel can be set to a desired value; however, the stable operating pressure normally differs slightly from this input.
A. Mach Number Validation

The test section static pressure, stagnation pressure and temperature are all sampled at 1000 Hz, averaged every ten samples and stored in a CSV file. A mean value was taken for stable period in which five traverse positions could be recorded and used as a measured value. The mean atmospheric pressure on the testing days was taken as 96 kPa from a barometer in the SSWT. The measured Pitot pressure is a mean value taken over the time of the stable flow.

Previous studies have used static pressure relationships for calculating the Mach number in the flow (Perry 1999; Hughes 2000). Firstly the isentropic relationship between the stagnation pressure and the free stream static pressure (Anderson 2007):

\[
\frac{p_0}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma - 1}}
\]

The Rayleigh Pitot tube formula was also used to take into account the (ideally) normal shock formed in front of the Pitot probe. It is a ratio of the Pitot pressure and the free stream static pressure (Anderson 2007):

\[
\frac{p_{02}}{p_1} = \left(\frac{(\gamma + 1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma - 1)}\right)^{\frac{\gamma}{\gamma - 1}} \left(\frac{1 + \gamma + 2\gamma M_1^2}{\gamma + 1}\right)
\]

It has been identified that these relations do not give reasonable results when solved for the free stream Mach number and a new relation was identified as more reliable (Janicke 2002). This variation of the Rayleigh Pitot tube formula expresses a ratio of the Pitot tube pressure and the stagnation pressure (Liepmann and Roshko 1957):

\[
\frac{p_0}{p_{02}} = \left(\frac{2\gamma}{\gamma + 1} M_1^2 - \frac{\gamma - 1}{\gamma + 1}\right)^{\frac{1}{\gamma - 1}} \left(\frac{1 + \gamma - 1 M_1^2}{\gamma + 1 M_1^2}\right)^{\frac{\gamma}{\gamma - 1}}
\]

The static pressure measurements taken were found to be extremely unreliable and low in this study, such that they were discounted from the calculations. Previous works found that consistent static pressure measurements were unreliably high and produced unreasonable Mach number values (Janicke 2002). Since the static pressure results were discarded for this thesis, such a comparison could not be made. It is assumed that the expansion of the flow from the stagnation chamber to the test section is isentropic and, therefore, the stagnation pressure will remain unchanged.

Table 1 shows the data collected in the validation; the Mach number was calculated from the Rayleigh Pitot tube formula variant used in MATLAB.

<table>
<thead>
<tr>
<th>Mach 2 liner (top)</th>
<th>Stagnation pressure (kPa)</th>
<th>Measured</th>
<th>Measured Pitot pressure (kPa)</th>
<th>Mach number</th>
</tr>
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<tbody>
<tr>
<td>Set (gauge)</td>
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<td></td>
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<td>200</td>
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<td>347</td>
<td>2.07</td>
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<table>
<thead>
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<th>Mach 3 liner (bottom)</th>
<th>Stagnation pressure (kPa)</th>
<th>Measured</th>
<th>Measured Pitot pressure (kPa)</th>
<th>Mach number</th>
</tr>
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<tbody>
<tr>
<td>Set (gauge)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>325</td>
<td>95</td>
<td>3.15</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>398</td>
<td>113</td>
<td>3.17</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>492</td>
<td>137</td>
<td>3.19</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Mach numbers calculated from pressure measurements.

The Mach numbers above are a reasonable result and correlate well with Janicke (2002). A slight variation in the Mach number with stagnation pressure is most obvious in the Mach 3 liner. This variation is within ±1% of the mean value; this could easily be accounted for as a cumulative error from the preceding averaging.

The mean values of M=2.07 and M=3.17 were determined for the Mach 2 and Mach 3 liner respectively. Although these will be used for subsequent calculations, the two flow regions will still be labeled as Mach 2 and Mach 3 for ease of reference.
B. Downstream Conditions

Figure 3, in Section II, shows a circular edge schlieren image of the asymmetrical flow studied. Both flow regions are generated by an expansion from the same stagnation pressure, hence the Mach 2 flow is characterised by a higher static pressure. This pressure differential results in the flow downstream of the trailing edge to be deflected downwards towards the Mach 3 flow. The Mach 3 flow experiences an oblique shock as it transits downstream; whereas the Mach 2 flow passes through an expansion fan before encountering a recompression shock. Janicke (2002) states that this shock causes only a small total pressure loss due to the splitter plate’s finite trailing edge. The same assumption is made for the following calculations of the downstream conditions.

1. Mach 2 flow properties

Using images such as the one above for each stagnation pressure, a number of important geometries were obtained. These demonstrated very little variation over the different stagnation pressures and the results are given in Table 2.

The meaning of the expansion geometries is made clear by Fig. 24; where θ is the wake deflection angle, μ₁ is the forward Mach angle and μ₂ is the rear Mach angle.

The equations used in relation to the expansion are the Prandtl-Meyer equations (Anderson 2007):

\[
v(M) = \sqrt{\frac{y + 1}{y - 1}} \tan^{-1} \left( \sqrt{\frac{y + 1}{y - 1}} \left( M^2 - 1 \right) - \tan^{-1} \sqrt{M^2 - 1} \right)
\]

\[
\theta = v(M_2) - v(M_1)
\]

And the Mach angle equation (Bertin 2002):

\[
\mu = \sin^{-1} \left( \frac{1}{M} \right)
\]

Firstly, the measured forward Mach angle was used to find a Prandtl-Meyer function, ν(M₁), and the subsequent value of ν(M₂) used to find the Mach number after the expansion fan. Secondly, the previously calculated M=2.07 was used as a starting point and the Prandtl-Meyer function solved for the downstream Mach number. The results are collated in Table 3; Prandtl Meyer function tables were sourced from Anderson (2007).
2. **Mach 3 flow properties**

The θ-β-M relation was used to determine the flow characteristics on the Mach 3 side of the flow (Anderson 2007):

\[ \tan(\theta) = 2\cot(\beta) \times \frac{M_1^2 \sin^2(\beta) - 1}{M_1^2(y + \cos(2\beta)) + 2} \]

Where \( \theta \) remains the wake deflection angle and \( \beta \) is the angle of the oblique shockwave. Other relationships used were are written below (Bertin 2002). The subscript \( n \) denotes a component normal to the oblique shockwave.

\[ M_{n,1} = M_1 \sin(\beta) \]
\[ M_{n,2}^2 = \frac{1 + (\frac{y-1}{2})M_{n,1}^2}{\gamma M_{n,2}^2 - (\frac{y-1}{2})} \]
\[ M_2 = \frac{M_{n,2}}{\sin((\beta - \theta))} \]

As with the Mach 2 flow, the downstream Mach number was calculated using two different items of acquired data. Table 4 below compares the two Mach numbers derived from the measured shock angle and the measured Mach number.

<table>
<thead>
<tr>
<th>Measured oblique shock angle</th>
<th>Measured Mach number</th>
<th>Calculated Mach number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured Mach number</td>
<td>2.07</td>
<td>Calculated forward Mach line angle 29°</td>
</tr>
<tr>
<td>Downstream Mach number</td>
<td>2.66</td>
<td>Downstream Mach number 2.53</td>
</tr>
</tbody>
</table>

**Table 3. Comparison of the Mach 2 flow properties.**

<table>
<thead>
<tr>
<th>Measured oblique shock angle</th>
<th>Measured Mach number</th>
<th>Calculated Mach number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured Mach number</td>
<td>3.17</td>
<td>Calculated oblique shock angle 27°</td>
</tr>
<tr>
<td>Downstream Mach number</td>
<td>2.49</td>
<td>Downstream Mach number 2.63</td>
</tr>
</tbody>
</table>

**Table 4. Comparison of Mach 3 flow properties.**

3. **Discussion**

The measurement of angles on the schlieren diagram, especially given the finite thickness of the key flow characteristics, is difficult to complete with an error less than ±8%. It can be seen in the tables above that the angles calculated from the measured Mach number are within the limit of error. The measured and calculated Mach numbers have a difference of ±6.3% or less; the measured values correlate well with previous theses. The measured Mach numbers and downstream flow properties calculated from them will be used for the remaining work.

4. **Relationships for future use**

The following relationships were used to express the wake flow conditions in terms of the stagnation conditions (Anderson 2007):

\[ \frac{p_0}{p} = \left(1 + \frac{y-1}{2}M^2\right)^\frac{y}{y-1} \]
\[ \frac{T_0}{T} = 1 + \frac{y-1}{2}M^2 \]
\[ p_2 = \rho_2RT_2 \]

The final relationships derived are shown in Table 5.
C. Reynolds Number

The Reynolds number can indicate whether a flow has remained laminar or transitioned to turbulent. The dynamic viscosity, \( \mu \), in this parameter is a function of temperature. In the case of the SSWT the temperature of the air in the stagnation chamber is slightly below the atmospheric temperature. A more accurate value of dynamic viscosity was calculated using Sutherland’s Law (Anderson 2007):

\[
\mu_0 = \frac{\mu_{\text{ref}}}{T_{\text{ref}}} \left( \frac{T_0}{T_{\text{ref}}} \right)^{\frac{3}{2}} \left( \frac{T_{\text{ref}} + 110}{T_0 + 110} \right)
\]

The reference values used were those of standard sea level air and the stagnation temperature was a mean value taken from the validation measurements. The values used and calculated are summarised in Table 6.

\[
\begin{array}{c|c}
\mu_{\text{ref}} & 1.7894 \times 10^{-5} \text{ kg.m}^{-1}.\text{s}^{-1} \\
T_{\text{ref}} & 288.16 \text{ K} \\
T_0 & 281.2 \text{ K} \\
\mu_0 & 1.76 \times 10^{-5} \text{ kg.m}^{-1}.\text{s}^{-1}
\end{array}
\]

Table 6. Values used in Sutherland’s Law relation.

The local speed of sound upstream of the trailing edge is given by the following expression (Bertin 2002):

\[
a_1 = \sqrt{RT_1}
\]

The Reynolds number for the trailing edge of the splitter plate was calculated for the very lowest likely total pressure in the reservoir, 295 kPa, and the highest total pressure recorded in the validation, 505 kPa. The Reynolds number for the Mach 2 side of the trailing edge was between 10.8x10^6 and 18.5x10^6; the Mach 3 side of the trailing edge was between 3.89x10^6 and 6.67x10^6. The lowest total pressure in the validation experiments was 311 kPa; 295 kPa is derived from the ideal set gauge pressure case (200 kPa) and the lower limit of atmospheric pressure (95 kPa).

The Reynolds number on either side of the splitter plate was also estimated using the following relation derived by Magi (1990):

\[
\frac{Re}{m} = (2835.4 - 1.6517c_0)(9.2852T_0 + 1995.7) \left( \frac{p_0}{T_0^2} \right)
\]

The results from this relationship suggest that the Reynolds number ranges from 19.6x10^6 to 33.5x10^6 for increasing total pressure. These values seem exceedingly high and the first, and lengthier, method of calculating Reynolds number was preferred for use in this study. Transition to turbulent flow should be complete by a Reynolds number of 3x10^6 (Munson, Young et al. 2006).

D. Boundary Layer Validation

From the schlieren visualisation: Mach 2 boundary layer is 5–6 mm, Mach 3 boundary layer is 6.5–7 mm thick. This correlates well with the turbulent boundary layers estimated at 5.5 mm and 6.5 mm respectively in Janicke’s study. Crocco and Lees (1952) produced the equation below can be used to describe a fully turbulent boundary layer.

\[
\frac{\delta}{m} = \frac{0.037}{(Re_d)^{1/4}} \left( \frac{C_{fM}}{C_{f1}} \right) \left( \frac{1}{\delta^{**}} \right)
\]

<table>
<thead>
<tr>
<th>Mach 2 liner side</th>
<th>Mach 3 liner side</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_2 = 0.0559p_0 )</td>
<td>( p_2 = 0.0478p_0 )</td>
</tr>
<tr>
<td>( T_2 = 0.439T_0 )</td>
<td>( T_2 = 0.420T_0 )</td>
</tr>
<tr>
<td>( \rho_2 = \frac{p_0}{2254T_0} )</td>
<td>( \rho_2 = \frac{p_0}{2522T_0} )</td>
</tr>
</tbody>
</table>

Table 5. Final relationships between the stagnation and wake conditions.
Using interpolated values for the thickness ratios and the compressibility correction (Crocco and Lees 1952), alternative theoretical values for the boundary layer thickness were obtained in Appendix F. These theoretical thicknesses varied between 6.6–9.3 mm.

VI. Visualisation Results

A. Trailing Edge Vortices

1. Recording rate of 500,000 frames per second

The images shown in Fig. 25–27 were recorded at 500,000 frames per second. It is difficult to see the evolution of trailing edge vortices from these images, especially in the upper shear layer; however, the video obtained when these still frames are played together provides a good deal more clarity.

The videos show that upper shear layer vortices appear to dominate in the near wake; whereas the lower shear layer shedding is inhibited by the oblique shockwave. It is possible that the apparent domination of upper layer vortices is because of their unusual interaction with the recompression shock.

From observation of the videos taken at different stagnation pressures, it appears that vortex shedding is suppressed with increasing stagnation pressure— or higher Reynolds number. This judgment is made from observing the vortices shed in the upper shear layer as changes in their behaviour were far more obvious than the suppressed vortices of the lower mixing layer.

The previous study in this field by Janicke (2002) indicated that the presence of vortices in the lower half of the shear layer could not be verified and that they were not prominent in that region. The circular knife edge videos obtained in this study do appear to reveal the shedding of vortices on the lower side, although these seem to be far less dominant than the upper vortices. Furthermore, these vortices do appear to propagate downstream in the same manner as those in the upper shear layer.

2. Recording rate of 250,000 frames per second

The images shown in Fig. 28–30 were recorded at 250,000 frames per second. This recording speed was not originally intended for use, but it was first employed accidentally and the images found to be of value. The higher speed videos showed the upper shear layer vortices to be strongest in their initial shedding stage but, although they did appear to propagate, did not provide much information on their continued behaviour.
downstream. The 250 kfps images show that the lower shear layer vortices do propagate downstream and their apparent large-scale entrainment of freestream flow is more effective than the upper shear layer vortices.

![Figure 28. Circular knife edge at 200 kPa; 250 kfps.](image1.jpg)

![Figure 29. Circular knife edge at 300 kPa; 250 kfps.](image2.jpg)

The upper mixing layer vortices exhibit similar shedding behavior to that shown in the higher speed images; however, the downstream behavior of these vortices is significantly more subdued than the vortices in the lower half of the shear layer.

The apparent dominance of the vortices in the upper shear layer may be due to their interaction with the Prandtl-Meyer expansion fan on the Mach 2 side of the flow.

A suppression of vortex shedding was observed of the upper shear layer for increasing stagnation pressures. This observation is validated by the vortex behaviour in the lower shear layer; that is, vortices are suppressed with increasing Reynolds number.

B. Inclined Structures

The schlieren setup used to visualise the inclined structures in the flow included a straight knife edge cut-off. Previous studies had used a vertical knife edge to visualise the inclined structures, although the Janicke (2002) inclined the knife edge by 11° to account for the deflection of the wake. The structures themselves are inclined at 45°–60° from the plate surface or wake centerline. Based on the measured wake deflection of 11° and an approximate 45° inclination of the structures, the knife edge was inclined at 35° to the horizontal axis extending from the splitter plate.

1. Recording rate of 500,000 frames per second

The images in Fig. 31 and Fig. 32 were recorded at 500,000 frames per second using the inclined knife edge cut-off. The setup of the schlieren system to visualise the structures in the wake have sacrificed the clarity of structures present in the boundary layer, although striations at approximately 45° can still be identified. The inclined structures do appear more clearly at a higher stagnation pressure, shown in Fig. 32.

Within the shear layer, a structured pattern at 45°–60° to the wake centreline is evident. It appears that the inclined pattern in the boundary layer is recreated in the shear layer. No images were taken to highlight the lower half of the shear layer; however, these have been done previously and demonstrate the same transition of these structures from the boundary layer to the shear layer. The inclined structures also appear to extend from the wake centreline to the edge of the shear layer, as they behave in the boundary layer, despite the increased thickness of the upper shear layer compared to the Mach 2 boundary layer.
2. **Recording rate of 250,000 frames per second**

   The images in Fig. 33 and Fig. 34 were recorded at 250,000 frames per second. The pattern of inclined structures can also be seen here, although they appear in greater clarity at a higher stagnation pressure. The structures in the shear layer appear to be inclined at a slightly lower angle to the wake centreline at higher stagnation pressures, lending evidence to the explanation that the hairpin vortices elongate with increasing Reynolds number (Head and Bandyopadhyay 1981).

3. **Existence in the boundary layer and shear layer**

   The visualisation demonstrates that inclined structures do appear in both the boundary layer and shear layer. It is not possible to verify from the images, or the video, if the structures somehow propagate from the boundary layer to the shear layer or immediately form in the shear layer at the trailing edge because that layer is turbulent.

**VII. Frequency Analysis**

**A. Base Signals**

Data was collected by the transducer when there was no flow passing through the test section in order to identify the noise present in the signal and discount it from the live results. This frequency profile also served as a basis against which the live results could be compared and dominant frequencies generated by the flow easily recognized.
1. No compressor

Figure 35 shows the FFT output when there is no flow present and the compressor is turned off. One of the lowest frequencies detected was approximately 50 Hz; this is likely to be noise from the alternating current in the signal. Other low frequencies were detected in order of 10, 10^2 and 10^3 Hz. All of these noise frequencies are of lower relative amplitude than the low frequencies found later in the saw-tooth frequency analysis. The dominant frequencies in this noise have a transformed amplitude in the order 10^{-4}.

The dominant frequency in the range shown is 92.27 kHz is found. Another dominant frequency of 32.13 kHz is found closer to the frequency region this study is interested in. The standout frequencies seem to be rough multiples of approximately 30 kHz, suggesting a phase repetition throughout the signal.

2. Compressor running

Figure 36 shows the FFT of the signal when there is no flow through the test section but the SSWT compressor is running. The frequency pattern for the range observed is almost identical, as is the amplitude order. This similarity was also found by Perry (1999), suggesting that the operation of the compressor has a negligible impact on the signal noise.

The dominant frequency was calculated to be 92.29 kHz. Although much lower in transformed amplitude, a spike at 31.63 kHz was also detected in the frequency range of interest.

B. Plain Trailing Edge Frequencies

Figure 37 displays the FFT of a signal 130 mm downstream of the plain trailing edge, at 200 kPa. The amplitude of the signal is the same order as the noise shown in the first two plots. Note the relative size of the 90 kHz spike on the right hand side. The dominant frequencies for the symmetric wake were approximately 22 kHz (Perry 1999), and so frequencies of less than 10 kHz were discarded. This was the same approach taken by Perry (1999) to remove the low frequency, high amplitude noise from the analysis found to occur in the base signals.

Figure 36 shows signal noise generated by the flow dominates the frequency region of 10–40 kHz. The largest frequency spike in this range is consistently around 31 kHz for all three stagnation pressures. The
transform amplitude of the 31 kHz spike here is very similar to the amplitude of the 31 kHz noise spike in the two base signal plots. By comparing Fig. 37 to the base signal plots, much of the noise in the 10–40 kHz region can be attributed to the flow, but there is no definite spike greater than the transform amplitude of the noise.

The frequency analysis for the plain trailing edge did not produce any definitive results. The frequency analysis for the saw tooth edge occurred in an experimental block some time later due to other work in the SSWT. Positioning the probe 130 mm downstream was difficult to ensure the probe was in the optimum position to detect the disturbances.

C. Saw-tooth Trailing Edge Frequencies

The frequency analysis for the saw-tooth edge also had little success at 130 mm downstream position. No dominant frequency could be distinguished from the noise. It was possible that, in the asymmetric wake, 130 mm is too far downstream to detect the vortices with the equipment available. Alternatively, the frequency at this position, for the asymmetric wake, may coincide with a dominant noise spike. Moving the transducer upstream offered the possibility being in a position where the vortices where stronger and easier to detect. Also, the vortices closer to the trailing edge should exhibit a higher frequency (Section II-B) that would separate a dominant frequency generated by the flow from the 31 kHz noise spike.

The probe for the saw-tooth frequency analysis was brought forward to a downstream position of 22.5 mm. For the vortices in the upper half of the wake the probe was vertically positioned approximately 4 mm below the trailing edge; 15 mm for vortices in the lower half of the wake.

1. Upper half of the trailing edge wake

Figure 38 shows the result of changing the downstream position. Note that the transform amplitude on the vertical axis is non-dimensionalised. The dominant frequency for this signal is approximately 27.9 kHz; this is

![Figure 38. The FFT of the signal 22.5 mm downstream from the saw-tooth trailing edge, at 300 kPa, over a frequency range of 10–250 kHz.](image-url)
similar to the dominant range in the plain trailing edge FFT and the 31 kHz noise spike, but note the relative amplitude of the 90 kHz noise spike on this plot. Contrary to the previous plots, the 90 kHz noise spike is less than 15% of the transform amplitude of the dominant frequency detected in the saw-tooth FFT; it follows that the 31 kHz noise spike is many times smaller in amplitude than the computed peak. This indicates that the dominant frequency detected has been generated by the flow, and is not a manifestation of noise.

Several frequency profiles were generated for each stagnation pressure and the average value of the dominant frequency was investigated. The approximate, mean dominant frequency was: 28.0 kHz at 200 kPa; 28.1 kHz at 300 kPa; and 28.3 kHz at 400 kPa. A trend is apparent where the dominant frequency is increased by rising stagnation pressure, or increasing Reynolds number.

2. Lower half of the trailing edge wake

Due to the apparent difference in the vortex shedding between the upper and lower shear layer, the transducer was also placed on the lower side of the wake to determine if a shedding frequency could be found and if it was appreciably different. Figure 39 is indicative of the other FFT results for the lower shear layer. There is a broad frequency range, specifically 20–30 kHz, which makes it difficult to validate the calculated dominant frequency.

The 200 kPa condition returned a dominant frequency on the lower mixing side of 18.6 Hz. The stand-out frequencies in the 300 kPa condition ranged between 22.9–28.3 kHz; and the 400 kPa flow returned an average of around 29.0 kHz. This is indicative of an upwards trend with increasing stagnation pressure; however, this would make the frequency jumps between stagnation pressures much higher than observed in the upper half of the wake, or by Perry (2009).

The lower shear layer results were not very consistent and did not offer an obvious fundamental frequency. There was no conclusive evidence gathered that the vortices on the lower shear side in an asymmetric flow have an appreciably different shedding frequency. In fact, no clear shedding frequency could be obtained from the lower shear layer; although the information above suggests a fundamental behavior close to that identified in the upper mixing layer. It can be observed in Fig. 39 that the range of high amplitude frequencies is much larger than that observed in Fig. 38. It is unclear what is causing this wide cluster of peaks and why it is so much more obvious in the lower half of the wake. Poor placement of the probe is a possibility, but it is unlikely that such an occurrence would be repeated several times for three different stagnation temperatures. It is possible that a downstream distance of 22.5 mm is too close to the trailing edge and the wake has not fully developed at the location of measurement.
VIII. Observed Effects of Trailing Edge Geometry

A. Convective Mach Number

The Convective Mach number is a constant property of a steady state shear flow studied in this thesis, describing the compressibility of the flow. The method of calculating the Convective Mach number for this asymmetrical flow was identical to Janicke (2002) and described in Section II-A. The Convective Mach numbers for either side of the wake are presented in Table 7, with Janicke’s 2002 results for comparison.

<table>
<thead>
<tr>
<th>Year of study</th>
<th>Post-trailing edge $M_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>0.0183, -0.0224</td>
</tr>
<tr>
<td>2009</td>
<td>0.0422, -0.0422</td>
</tr>
</tbody>
</table>

Table 7. Comparison of Convective Mach numbers.

B. Visualisation of Trailing Edge Vortices

There was no observation of any significant differences between the large-scale vortex behaviour of the two geometries available from the schlieren imagery recorded. The cyclic interaction between the expansion fan/recompression shock and the shear layer, visible with the video, occurred with both geometries with no discernable difference in the strength of that interaction—although a difference in the steady interaction is discussed below.

As with the plain trailing edge, the formation of vortices on the lower side of the wake was apparent, especially from the videos. There was no observable difference of this behaviour between geometries.

C. Visualisation of Inclined Structures

From the still images, the trailing edge geometry did not seem to affect the inclination angle or appearance of the inclined structures in the shear layer. From the short videos, there did not appear to be any difference in the transition of the inclined structures through the shear layer from that which was observed for the plain trailing edge.

The formation of the shear layer at the trailing edge did, however, highlight a difference in the saw-tooth geometry. Figure 47 highlights the upper shear layer at its formation. The thickness of this region appears to be three or four times that of the equivalent region behind a plain trailing edge. It is possible that the expansion fan interacts with the saw-tooth geometry such that the formation of the shear layer is not as significantly suppressed. There does not appear to be any difference in the interaction of the lower shear layer and the oblique shockwave.

D. Velocity Profiles

The following velocity profiles were derived from the traversing Pitot probe surveys. These are shown in Fig. 40 and Fig. 41. The lower half of the shear layer appears to be characterised by a steeper velocity gradient than the upper half. This can be explained by both the faster freestream speed on the lower side and the smaller thickness of the shear layer below the centreline, seen clearly in Fig. 31.
1. **Plain trailing edge**

![Velocity measurements of wake by traversing pitot probe at 300 kPa stagnation pressure]

Figure 40. Plain trailing edge velocity profile.

2. **Saw-tooth trailing edge**

![Velocity measurements of wake by traversing pitot probe at 300 kPa stagnation pressure]

Figure 41. Saw-tooth trailing edge velocity profile.

E. **Wake Half-width Growth**

The wake half-width provides a means of measuring the growth of the wake from the velocity profile information collected. A higher rate of wake growth indicates a greater degree of entrainment.

The maximum defect, $W_0$, is defined graphically in Fig. 42, corresponding to the minimum velocity in the profile. The wake half-width, $b$, is also defined in Fig. 42. The wake half-width is calculated at each downstream position for a trailing edge geometry; the rate of change in this value yields to wake half-width growth.

The growth rate for the plain trailing edge was observed to be 0.055; the saw-tooth trailing edge growth rate was 0.072.

F. **Similarity Profiles**

Previous studies have shown that a wake will develop local similarity at some distance downstream from a disturbance. Prior to this similarity is
a developmental region of the shear layer where it adjusts to an equilibrium state.

In order to obtain similarity profiles for the two geometries, the local velocity defect was normalised by the maximum defect in the profile; the vertical distance was normalised by the wake half-width growth. For both geometries a reasonable similarity was observed from $x=11.5$ mm to $x=76$ mm, shown in Fig. 43 and Fig. 44.

G. Wake Growth From Visualisation

Figure 44. Saw-tooth trailing edge similarity profile.

Figure 45. Shear layer bounded by an approximate boundary, to measure wake growth from visualisation.
1. **Plain trailing edge**

The wake growth rate was measured from an adjusted inclined knife edge schlieren images, an example shown in Fig. 45. The inclined knife edge technique gave the best clarity for the transverse density gradients observed in the flow. The mean growth rate for the plain trailing edge was found to be 0.02. This is slightly lower than the value of 0.03 obtained by Janicke (2002).

2. **Saw-tooth trailing edge**

Using the same technique as above, a wake growth rate of 0.03 (rounded from 0.027) was obtained for the saw-tooth trailing edge. Although this is an increase on the rate found for the plain trailing edge, it is difficult to declare this difference significant due to the error in this visual method.

**IX. Conclusion**

The new, high speed camera is a very valuable tool for visualisation. The ability to capture multiple frames for consecutive playback allows the asymmetric flow to be presented more intuitively for study and permits more informed observation of unsteady behaviour.

The flow visualisation conducted suggests that vortices are shed in the lower wake. The interaction between the expansion-fan/recompression shock and upper shear layer is considerably stronger than the interaction between the oblique shockwave and the lower shear layer.

The inclined structures in the boundary layer and asymmetric shear layers found by Janicke (2002) are again shown here. While this may confirm the existence of hairpin vortices in both regions, there is no evidence observed in this study that the structures are convected, in the traditional sense, from the boundary layer to the wake. These vortices are, by their nature, subject to the creative and destructive forces that exist in a turbulent equilibrium (Theodorsen 1955). Rather than being unaffected by the trailing edge flow behaviour, it is possible that they are destroyed and reform in the turbulent wake.

A dominant frequency could not be obtained for the plain trailing edge geometry. A series of dominant frequencies were obtained for the saw-tooth trailing, albeit at a much shorter distance from the trailing edge. A broader frequency range of similar magnitude was found for the lower half of the wake. This suggests that vortices of similar frequency are shed in the lower half of the wake.

The findings of Janicke (2002) regarding the effect of trailing edge geometry on wake growth were supported.

**X. Recommendations**

The new approach to schlieren visualisation available with the Shimadzu HPV-1 camera should more thoroughly explored to find the most effective specifications for studying the supersonic wake. Although the new angle of the inclined knife edge did produce useful results, a project more dedicated to visualisation may offer scope to investigate different cut-off configurations.

A future frequency analysis should attempt to gain results at a point further downstream of the splitter plate trailing edge. Little success was had at a distance of 130 mm in this project but it is unclear if that was due to the flow properties or some element of the method. It is possible that a distance of 22.5 mm did not allow the wake enough time to develop for a more conclusive result.

**Acknowledgements**

I would like to thank Associate Professor Sudhir Gai for his guidance in the project and extensive knowledge in the field. My thanks also to Associate Professor Harald Kleine for his expertise in schlieren visualisation and the operation of the HPV-1 camera. I am indebted to Mr Michael Jones for his many hours of assistance over the extent of the project. Mr Andrew Roberts, who wrote the new code for the stepper motor and assisted in the calibration of the transducer, was invaluable in establishing the data acquisition system. I also thank Mr Simon Parcell for his time in running the SSWT for my experiments.

**References**


