Pilot study for the development of a visualisation system for large scale blast waves

B. Pavlovic*

University of New South Wales at the Australian Defence Force Academy, Canberra

This report describes a study on the suitability of a smoke screen visualisation system for large scale blast waves utilizing current Background Oriented Schlieren (BOS) techniques. It is to be evaluated in a laboratory-scale experiment whether accurate flow measurements of a blast wave can be obtained by high-speed photographer without the need for a large scale schlieren optical system. The success of this study can be judged based on the similarity between the results from testing with standard visualisation techniques for an experiment run under identical conditions. It was found that a smoke screen background is, indeed, suitable to detect a blast wave front, but it also became obvious that the characteristics of the smoke screen have to be carefully chosen to obtain sufficient visibility of the wave.

Nomenclature

\[ a = \text{speed of sound} \]
\[ t_s = \text{time} \]
\[ p_s = \text{ambient pressure} \]
\[ p_o = \text{high pressure section} \]
\[ \rho = \text{density} \]
\[ s = \text{distance} \]
\[ v = \text{velocity} \]
\[ M = \text{Mach number} \]
\[ \gamma = \text{adiabatic index, 1.4 for air} \]
\[ R = \text{universal gas constant} \]
\[ n = \text{refractive index} \]
\[ S = \text{Sensitivity} \]
\[ L = \text{distance between camera and background} \]
\[ t = \text{distance between camera and test section} \]
\[ \epsilon = \text{angle of deflection} \]
\[ f = \text{focal length} \]
\[ \Delta ij = \text{detected distortion} \]

I. Introduction

Blast waves caused by large-scale explosions can sometimes be seen from the distortion of the background which is a result of the change in the density caused by the wave. The background of a large scale explosion generally consists of landscape and sky so there may not be enough detail to identify a clear change as the blast wave passes.

Density sensitive visualisation techniques, such as a schlieren or shadowgraphs optical system allow us to see the change in the density of a flow. The differences in density, caused for example by a shock wave, can be made visible which allows us to quantitatively analyse the wave front typically in a laboratory environment. This is typically done in a laboratory environment as the necessary optical components of the setup limit the field of view to areas of usually less than 500mm in diameter. There are a number of variations of each technique but the concept remains the same, namely recording the refractive index differences as a result of density variations.

Background Oriented Schlieren (BOS) is a flow visualisation technique that can be used more simply in the large scale to visualise shock waves. The physical principle used is the bending of light rays due to inhomogeneities in the medium caused by the moving shock front and recording them using a high-speed camera in the direct line of sight. An avenue for improvement lies in the control we have over the background pattern. This pilot study will use laboratory-scale experiments with a different artificial background building upon previous studies on BOS. The goal is to test, on a laboratory-scale, a background that can could be used on a large scale.

*PLTOFF, School of Engineering and Information Technology, ZEIT4500/4501
II. Project Outline

The aim of this project is to extend on the current applications of visualisation systems that can be used to effectively track the propagation of a large-scale blast wave. The primary requirement is that the background has enough detail in order to view the distortion due to the density gradient caused by the shock wave at all points during propagation. It is suggested that a smoke screen background will be suitable for the purpose. Such a smoke screen background is relatively easy to generate, it can be made several tens of meters large, and the speed of the rising smoke is typically much lower than that of the observed shock wave making it effectively stationary.

This study will investigate if laboratory-scale smoke generated background will be an adequate background to effectively visualise a propagating shock wave in this case, an open-end shock tube to simulate a blast wave.

This project is broken up into three major components:

1. Baseline testing is carried out using the typical laboratory schlieren visualisation technique as this is the most effective way of viewing the passing shock wave. Using the images obtained from these experiments, the shock front is readily identifiable and tracked in the form of a distance-vs.-time graph and ultimately a measurement of the overpressure of the travelling shock. These baseline tests are carried out multiple times to ensure and prove reproducibility of the experiment.
2. Standard BOS tests incorporate a randomly generated dot background and are directly filmed with a high-speed camera. With the use of image processing and a MATLAB program the wave front will be tracked and compared with the results of part 1.
3. A smoke screen is generated to provide a background that satisfies the criteria detailed in this report. The computational method detailed in step two is used to create an overpressure-vs.-distance plot to determine if the shock wave tracking is equivalent to that of the baseline tests. The expectation and proof of success will rely on the overpressure-vs.-distance plots for all the baseline test data and all BOS data to yield the same results.

III. Literature Review

A. Blast Waves
An expanding blast wave is normally produced by the rapid release of energy from a centered source [1]. A sphere of gas with high pressure and temperature expands rapidly in the ambient medium resulting in a shock wave propagating outward from the driving source. The shock front typically has the width of a few tenths of nanometers under ambient conditions. The shock induced overpressure and its duration are used to determine the potential damage from explosions [2].

The ratios of the fluid properties can be calculated from the shock Mach number \( M_0 \) using the Rankine-Hugoniot (R-H) relationships [3]. The ratio for pressure with relation to the Mach number is:

\[
P_0\frac{P_1}{P_0} = 1 + \left( \frac{2\gamma}{\gamma+1} \right) (M_0^2 - 1)
\]  

(1)

The strength of a shock wave decreases as it expands from the driving source. In order to plot the overpressure with respect to the distance from the point source, the location of the shock front is tracked and recorded in the form of a distance-vs.-time graph. An empirical relationship for the trajectory of a shock (with respect to time) has been used successfully in past studies and is known as the Dewey curve [4]. A curve fit is established with the equation 2, where A, B, C and D are the coefficients found from the raw data:

\[
s = A + Bat_s + C\ln(1 + at_s) + D\sqrt{(\ln(1 + at_s)}
\]  

(2)

Experimental data will provide the coefficients so that the above relation can then be differentiated to find the velocity with respect to distance. The velocity of the shock is directly related to Mach number which requires measuring of the temperature to determine the speed of sound on the day. This is then plotted corresponding to the relative distance recorded. Using the R-H equation above at equation 1 it is possible to plot the overpressure-vs.-distance profile for the shock. This profile can be used to compare the results of the visualisation technique developed in this study with the results obtained by the established methods.

B. Open-end Shock Tubes
A shock tube, in its simplest form, is a tube with a high pressure (driver section) and a low pressure section separated by a diaphragm. The flow is initiated by bursting the diaphragm which causes a discontinuity (shock wave) to propagate...
through the tube with a specific Mach number depending on the pressure difference. When the shock wave emanates from the open end into the surrounding atmosphere, it creates a complex flow field made up of the expanding shock wave and followed by a vortex ring and jet of gas exiting the tube [8]. The focus of this study is to track the expanding shock wave front, which is used to simulate a real blast wave.

By specifying the conditions in the high and low pressure sections, the shock Mach number is uniquely identified [2]. Combining the jump conditions across the shock and the isentropic relationship for the expansion into the high-pressure part yields the following equation for ratio of the high pressure section and the ambient air as a function of shock Mach number and gas properties [3]:

\[
\frac{p_4}{p_1} = \left[1 + \frac{2\gamma}{\gamma+1}(M_s^2 - 1) \right] \left[ \frac{1}{1 - \frac{\gamma - 1}{\gamma + 1} \left( \frac{M_s}{M_s^2} - 1 \right)} \right]^{\frac{2\gamma}{\gamma-1}}
\]

C. Visualisation Techniques

Flow visualisation techniques are used to provide optical data of the behaviour of fluids which are generally colourless and transparent and not visible to the naked eye. One of the most common types of density-sensitive flow visualisation techniques is the schlieren optical system. These methods use the phenomenon of the change of the refractive index of a light beam through differing densities, which are then recorded using (high-speed) photography.

Schlieren systems measure the deflection of a light beam that traverses the test section. The deflection is caused by a density gradient normal to the beam [5]. These deflected light beams are ‘cut off’ by a knife edge which results in a visual indication of the cause of the density change. This leads to a proportional change in illumination intensity on the recording device [6]. This method typically requires expensive and fragile optical equipment and a sensitive setup.

D. High Speed Photography

Modern digital camera technology has made it possible to visualise high-speed flows in single-image or time-resolved mode. Time-resolved visualisation captures a sequence of flow images at set intervals to demonstrate the evolution of changes of a flow field [5]. These images make it possible to track the shock front and other such flow field characteristics that cannot otherwise be seen. True time-resolved visualisation is capturing sequential images of the same experiment, rather than taking single images at different points of multiple experiments of the same nature. This reduces the risk of missing the highly transient intermediate stages of the flow [5]. This makes it possible to ‘track’ the shock front or pathlines which then can be further analysed.

Modern cameras also provide the ability to control the exposure time of the image. The exposure is the amount of light that reaches the recorded image which is determined by the available shutter speed and the lens aperture. Reducing the exposure time ‘freezes’ the rapid motion [5] of an object. Digital shutters are much faster than a mechanical shutter, which has also vastly improved the capability of high-speed photography.

The f-number of a camera is the ratio of the cameras’ focal length to the diameter of the aperture. The brightness of the projected image decreases with the square of the f-number [9]. This requires an increase in the exposure time to allow
for the illumination of the image to be at the same intensity. The depth of field of an image increases with the f-number which allows for more subjects in the image to be in focus.

E. Preceding Research in Background Oriented Schlieren Systems

The above visualisation techniques are useful on a small scale, but are not appropriate for large scale testing. The visualisation of such a large field of view with these techniques would require large optical components and screens which are difficult and expensive to manufacture [11]. A background of sufficient contrast and pattern that reveals the amount of distortion caused by the passing shock wave is a more efficient way than the more complicated visualisation techniques discussed above, especially in large scale tests. The basic premise includes comparison of the undistorted view with the distorted view, usually through a number of image processing algorithms to estimate per-pixel displacement vectors between two images [12].

A natural background of an outdoor test site can sometimes be used as such a background, subject to criteria of fine scale, randomness and contrast [8]. If the background does not meet these criteria, the accurate measurement of the distortion becomes difficult. This may occur when the natural background includes a clear blue sky, and therefore no changes can be identified from the lack of distinctive changes between the distorted and non-distorted images.

The flow causes inhomogeneities in the medium which will influence the refractive index [15]. The background distortion is directly related to the strength of the schlieren object as well as the optics and physical geometry of the experiment [9]. The sensitivity mostly depends on the focal length of the lens and the relative position of the object between the camera and background, as well as the smallest detectable shift in the imaging plane [10]. Figure 2 shows a typical schlieren method optical arrangement originally proposed by Hubert Schardin in 1942.

![Figure 2. Schardin’s Schlieren Method [10]](image)

The sensitivity is maximised by minimising the ratio t/L (the distances t and L are indicated in figure 2). This is limited by keeping both the background and the test section in reasonable focus, whilst maintaining the background sharp to enable identification of the distortion caused by the shock wave [10]. Other limiting factors are the capability of the high-speed digital camera, in particular the available pixel resolution. The sensitivity can be defined as the ratio of the detected change or pattern shift to the corresponding change in the angle of deflection [15] in the form of equation 4. These components are depicted in figure 2.

\[ S = \frac{\Delta ij}{\epsilon} = \frac{f(L - t)}{L} \]  

(4)

On a laboratory scale, computer-generated backgrounds have been developed that allow full control over the background pattern. This can prove to be more accurate in measurements of the distortion, however is not as convenient as using an already in place natural background. The first BOS system employed in this project is a computer-generated random dot pattern of sufficient randomness and contrast against which, after some image processing and manipulation, one is able to visually identify the blast wave.

Successfully identifying and tracking the wave front using a random dot pattern will allow application of the same algorithm using the smoke screen to prove if this is a viable background which can be translated to the large scale.

IV. Baseline Testing

A. Method

Baseline wave patterns are produced by an open-end shock tube using the typical schlieren visualisation technique. Time-resolved imaging allows us to track the front of the shock wave to obtain position-vs.-time data which can be used to calculate the pressure amplification as a function of the distance, or the radius of the shock wave.
The high pressure section of the shock tube is pressurized with 545kPa of air provided from the reservoir. The nozzle end is open to ambient pressure and temperature was recorded on the day (typically 95.6kPa and 294.15K). Once the desired pressure level in the high-pressure part has been reached the diaphragm separating the high-pressure section to the tube is punctured which results in a shock wave propagating through the tube. The set up for these experiments is depicted in figure 3 above.

The pressure transducers are piezoelectric sensors and will generate a signal when the shock passes over them. This signal is recorded on an oscilloscope. From the oscilloscope, a delay generator is activated and this unit sends signals to the light source and the camera to synchronize these two components and the event. The camera starts recording when the shock is reaching the end of the tube at which time the flash has reached full power. The velocity can be determined from finding the time taken for both pressure signals recorded by the oscilloscope. It is of note that the time is recorded from the same point on each signal, in this case the peak of the initial disturbance from the signal. The error from reading the values of the peak off the oscilloscope is ±0.2μs.

<table>
<thead>
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<tr>
<td>Time between frames</td>
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<td>Number of frames</td>
<td>100</td>
</tr>
<tr>
<td>F number</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Table 1. Camera setup for schlieren tests

Both methods for this experiment require high-speed photography as the shock wave passes through the test section with a transit time of less than 500μs. In this application, 100 frames were taken which will yield 100 data points.

B. Analysis

The velocity of the shock wave is recorded from the timing on the oscilloscope from when the shock passed the first pressure sensor to the second which is at a distance (s) of 60mm downstream. The temperature was measured to calculate the speed of sound on the day. The Mach number achieved under these conditions is 1.39.

The value of the obtained Mach number can be checked using the theoretical prediction from equation 3. The value of the pressure ratio from equation 3 using the recorded average Mach number is calculated to be 4.95. This is inconsistent with the actual p4/p1 ratio using the measured pressure differences, of 5.7. Iterating equation 3 to obtain this measured pressure ratio yields a Mach number of 1.44, or 495m/s. The difference calculated amounts to 3%.

MATLAB was used to determine the front of the shock from the time-resolved images. A reference length was incorporated in the filming by imaging an object placed at a known distance from the shock tube end. This allows one to obtain a pixel-to-length calibration to determine the distance of the shock front from the shock tube exit. The error for the location of the shock wave on the image is ±1 pixel. With the given setup, this corresponds to a physical difference of 0.62mm. An example of a frame recorded in these tests is depicted in figure 4.

The results were then evaluated by finding an appropriate curve fit for the graph in figure 5, the differential being velocity with respect to time. The curve fit for the model that was used was the form proposed by Dewey in equation 3. The curve fit is differentiated to obtain velocity and then converted to Mach number and plotted against the corresponding distance at that point in figure 6.

The graphical representation in figure 6 suggests a Mach number larger than 1.4 at distance = 0. This is because Dewey’s curve fit was developed for a spherical wave starting from a (small) charge. In this case, we have a non-spherical wave and even a partially straight wave for the first approx. 20
microseconds – the curve fit can only be expected to yield reasonable results once the straight shock segment has disappeared. Therefore, the Mach number-vs.-distance curve has been truncated to only include data that corresponds to a wave without any straight segment.

![Raw Data of Schlieren Tests](image)

**Figure 5.** *Direct measurement of the distance of the shock wave from the shock tube exit at time, t*

![Mach number](image)

**Figure 6.** *Decay in the Mach number with respect to the distance from the shock tube exit*

![Overpressure](image)

**Figure 7.** *Comparison of the overpressure from the three baseline schlieren tests*

This plot is the standard baseline to be compared with the next two parts of this study.

A limitation of the reproducibility of the tests is that the user has no control on how the diaphragm bursts. This is an accepted error and has negligible effects on the overall outcome but must be considered when determining how precise the behaviour is allowed to differ.
At the final frame at $t=0.00404\mu s$ and $s=0.150m$, the average overpressure was determined to be 1.055. These values are compared in table 2 to show the reproducibility of this technique. The corresponding error is determined by the following equation:

$$M = \frac{V}{a} = \frac{\Delta x \pm 0.4\text{mm}}{\Delta t \pm 0.5^\circ C}$$

Table 2. Reproducibility of tests: Overpressure

<table>
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<tr>
<th>Test</th>
<th>Ps/Po (at s=0.150m)</th>
<th>Average (Nominal) Error</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>1.035</td>
<td>1.9%</td>
</tr>
<tr>
<td>2</td>
<td>1.067</td>
<td>1.2%</td>
</tr>
<tr>
<td>3</td>
<td>1.063</td>
<td>0.8%</td>
</tr>
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V. Background Oriented Schlieren Tests

A. Method

The BOS tests were conducted in the lab similarly to the initial baseline testing. This time, a random dot generated background was used, and the high-speed camera was simply filming the event without the use of elaborate equipment that is required for standard schlieren visualisation techniques. As the distances between the camera, test section and background are important for the comparison with the smoke generated background, these details are recorded so they can be replicated. The set up can be seen in figure 8. The reduction in frame rate is due to the use of a different camera for the BOS technique as it is particularly important to maintain high resolution not achievable by the camera used in the baseline testing. The baseline tests used a resolution of 312x260 pixels while the BOS tests used 912x200 pixels.

![Figure 8. BOS laboratory setup](image)

A non-action, or ‘no flow’ test is filmed initially to determine the background noise caused by the camera. Slight, subtle frame-to-frame intensity changes generated by the camera itself when filming the random dot pattern can cause changes at a pixel level and therefore suggest that a flow is present. As this project is utilizing the changes caused at a pixel level, it is important for the first test to determine if such camera-induced changes can be reduced or removed so as not to cause false indications of movement.

To eradicate/determine noise resulting from ‘flickering’ amongst the pixels, the image subtraction technique is applied to the no flow images. This shows the internal changes (ie due to the camera or subtle movement) that are not relative to the flow event. The second reason for this test is to view the changes in intensity of each frame caused by the varying flash output over the given time. The test produced frames all of differing illumination of a certain rate which will be required to ‘even’ out, so that when image subtraction occurs again will not produce false indications where there is simply a change in illumination intensity not a change in the density. A MATLAB function was created to equate the difference in brightness from a reference image in order to equalize the flash intensity of each of the images.

B. Analysis

A total of three separate BOS experiments were carried out. The initial test using a pre printed random dot background of arbitrary size was used and the high speed camera on its standard settings. This resulted in noisy data that was requiring a large amount of post processing and computation. It was determined that this intensive process was inefficient and unnecessary and a second experiment was carried out to obtain better data. This involved the use of calculating the pixel size and determining from the distance in the setup, what size the dot pattern should be to have at most on dot imaged onto a single pixel. The pixel size was calculated using the known reference length within the
image and the number of pixels per length, determined to be 0.41mm per pixel. The sharpness of the reference length at the chosen f-number is within 10 pixels.

Performing a second BOS experiment with the alternate dot size more applicable to our specific setup, provided a clearer distinction in the intensity of the pixels as the image subtraction process was performed due to the removal of the ‘averaging’ of pixel values. There was still significant ‘blur’ from the test section.

Finally, changing the f-number (and corresponding exposure time due to limited lighting) allowed for the test section to also be more in focus, not just the background. It was found that whilst it is of utmost importance for the background to be sharp, it provided better images when the test object itself was also sharper.

From this point once the image has been cleaned up and the shock front is made obvious, the same MATLAB program and analysis of the results is carried out and compared to the baseline tests. Figure 10 shows the overpressure-vs.-distance of the BOS tests and the baseline tests.

<table>
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<th>Test 2</th>
<th>Test 3</th>
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<td>29,000fps</td>
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<td>Exposure time</td>
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<tr>
<td>Time between frames</td>
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</tr>
<tr>
<td>Number of frames</td>
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<td>10 (effective)</td>
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<td>F number</td>
<td>5.6</td>
<td>16</td>
<td>22</td>
</tr>
</tbody>
</table>

Table 3. BOS test camera setup

Post image processing is key in this method and the resulting images after subtracting from the no-flow image can be seen in figure 9. The wave front and subsequent vortex ring can be seen.

![Background Oriented Schlieren](image)

Figure 9: Background Oriented Schlieren

The BOS technique is limited by the pixel resolution based on the field of view and the camera capabilities. The image resolution influences the experimental error in the measurements of the shock position and velocity which limits the accuracy of pressure predictions [13]. This is evident in the image subtraction images with the ‘blur’ of the vortex ring.
and the subtle outline of the wave front. This blur was spread over approximately 10 pixels, which gives an error margin of +/- 2mm.

For the purpose of this study, the resulting overpressure-vs.-distance graph is within error limitations. The BOS plot is curve fit with significantly less data points due to the reduction in the frame rate to cater for the high resolution required and therefore the curve is not a smooth as the schlieren plot. From approximately s=0.030m the plots are almost identical. These results determine that the Background Oriented Schlieren follows the expected plot sufficiently to apply this method to part 3, the smoke background.

VI. Smoke Generated Background Experiments

The set up of the equipment and the camera is identical to the BOS tests in order for the analysis and MATLAB code to be applicable. A smoke screen is created with a uniform backdrop behind the screen so that the contrast created is solely from the smoke. The same experiment was carried out multiple times to ensure that the smoke covered sufficient area in the test section and relied heavily on the timing of the experiment due to the highly disordered movement of the smoke. The tests resulted in being able to see the movement caused by the passing shock wave which was a good indication that the distortion is sufficient to visualize the shock. In all tests, the smoke itself was seen to be essentially stationary as expected.

An example of the initial image obtained from these tests is seen in figure 11 (a). The same algorithm is applied for the post image processing that was carried out for the BOS tests for consistency. Figure 11 (b) shows the processed image (without the reduction of the noise) in which one can see the shock wave.

With the application of the MATLAB code with slight amendments to counteract for the noise, the images are able to be analysed and the output is the distance-vs.-time. The images are of low contrast and difficult to see by the human eye (figure 11 (c)), therefore the sensitivity of the MATLAB program had to be adjusted to be sure to pick up the right pixel which represented the shock wave which due to this lack of contrast reduced the intensity of the distortion significantly. Again the graphical representation of the Mach number with respect to the distance is obtained by differentiating this curve and converting the values for velocity to Mach number. The overpressure is calculated again using equation (1).

![Image](https://via.placeholder.com/150)

**Figure 11.** Smoke screen image (a) pre image processing (b) Initial processing to visualize the shock wave (c) post image processing removing excess noise

The resulting Mach number and corresponding overpressure (with respect to distance) is plotted and recorded in figure 12. The comparison of these graphical results determines the success of the smoke screen as a visualisation method for propagating shock waves.
The reduction of data points to 10% of those available for the standard schlieren technique results in a less reliable curve fit as seen in the random dot background experiments.

From this point forward, the overpressure for the random dot pattern and the smoke screen background are within 8% of the nominal values, but as can be seen from the minimum smoke error curve, the results are within acceptable limits from the variables incurred in this study. The error curve is derived from equation 5 as previously discussed as well as the limited sensitivity of the smoke screen images due to the loss of contrast. Similarly, both the random dot pattern and smoke screen incur a larger error in pixel location due to the blurred outline of the shock front which spans up to 10 pixels (±4.1mm).

The comparison data is between the random dot background and the smoke background, as these techniques both used the same camera that was better suited to maintain a high resolution. The plots follow the same shape curve which is encouraging that the results are as expected, and that this technique has in fact plotted the wave front.

VII. Conclusion

The outcome of this feasibility study is a strong indication that the background applied in this study has the characteristics required to be able to provide enough contrast to see a distinct distortion caused by the passing shock wave.

Multiple experiments were repeated in order to distinguish key variables and obtain clearer data that required less image processing. The image processing technique is ambiguous and further investigation to the ideal algorithm to produce clearer results should be conducted. The results have provided data which is difficult to analyse computationally and therefore some error is incurred in the exact location of the shock front. Further research would improve these results both regarding the image processing algorithm and the type of smoke generated to increase the contrast within the image and therefore precision of the overpressure curve.

The finite scope of this initial study has limited some key variables. The smoke colour, distribution system and physical layout of the entire system have had some restrictions, due to being conducted within a laboratory environment of set dimensions and limited ventilation. This study has shown that the distortion is detectable using this method and further testing using higher density smoke to provide more contrast can be used to solidify these results; however this pilot study has shown that the smoke is feasible to visualize the propagation of the shock wave.
References