Design and Analysis of Meta-materials

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This report summarises my project on mechanical tunable meta-materials working in the Terahertz region (0.1 - 10 THz). In many situations, terahertz energy is generated by impinging broadband mode-locked laser pulses into photo-conductive antennas, this meaning that the generated terahertz pulses cover a wide spectral range therefore it is important in many applications to isolate a portion of the available spectrum by using filters e.g. detection of the presence of a certain substance. In this project we propose tunable filters that could be tuned using both a planar and out of plane mechanical movement of the SRR’s of a range of 200GHz.

Nomenclature

- $h$: Thickness of homogeneous meta material, m
- $\varepsilon$: Electric permittivity, F m$^{-1}$
- $k$: Free space wave number, rad m$^{-1}$
- $m$: Mass of an electron, kg
- $n$: Refractive index
- $\mu$: Magnetic permeability, H m$^{-1}$
- $\omega$: Frequency, rad s$^{-1}$
- $Z$: Impedance, $\Omega$

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I. Introduction

META-materials as defined by Cai and Shalaev,\textsuperscript{1} are artificial sub-wavelength materials which maintain a homogeneous electromagnetic response expressed by macroscopic material parameters such as the effective electric permittivity. In this case the small dimensions of the material can be averaged to produce an effective magnetic permeability $\mu_{\text{eff}}$, electric permittivity $\epsilon_{\text{eff}}$, effective Impedence $Z$ and effective refractive index $n$. These parameters are essential when it comes to mediums and the electromagnetic spectrum, and in this case they can be defined by the design of the unit cell.

I.A. Split Ring Resonator (SRR)

In this report the SRR will be discussed as shown in Figure 1. This SRR is a horseshoe structure which is easy to manufacture and is defined by 4 parameters being the Width, Length, Gap Width and Gap Length. It also provides a an $\epsilon_{\text{eff}} > 0$ and a $\mu_{\text{eff}} < 0$. Each of these dimensions affect how the structure reacts with electromagnetic waves as can be seen in Figure 1 where the capacitance and inductance changes with every variation. The dimensions of the unit cell were $160\mu\text{m} \times 160\mu\text{m}$ with the SRR having the dimensions seen in Figure 1.

![Figure 1. Parameters of the unit cell and the equivalent circuit](image)

II. Model

THE simulations of a meta-material unit cell for this project were conducted in CST using a frequency solver with unit cell boundary condition. CST uses finite element methods to analyse the propagation of electromagnetic waves in complex structures. The unit cell boundary conditions virtually repeats the modelled structure periodically along the plane that contains the horseshoe structure. Figure 2 shows how a modelled unit cell will appear under unit cell boundary conditions.
This unit cell model places the SRR mentioned earlier on a substrate made of quartz which has a refractive index of 1.95 with the SRR being made of gold. Gold is modelled as a dispersive material and the whole structure is surrounded by air. With the dimensions shown in Figure 1, resonance was achieved at 1 THz which can be seen in Figure 3.

**II.1. Resonant frequency**

The SRR dimensions are characterised for resonance at a desired frequency, this is where the highest amount of electromagnetic activity occurs for the SRR and where the effective parameters show unexpected behaviour. The resonance of the SRR is determined by Eq. 1

\[ w_0 = \frac{1}{\sqrt{LC}} \]

and demonstrates how it is dependent on the capacitance and inductance of the SRR. Detecting the resonance is done by observing the S-parameters of the unit cell where the structure is assumed to infinitely periodic in the plane of the meta-material. At resonance the S-parameters show reflection via the S11 parameter and transmission via the S21 parameter. Looking at the phase of the S-parameters reveals that the phase transitions to positive from negative at resonance which is another method of resonance detection. The transmission and phase change can be observed in Figure 3 where it is noted there is no transmission at resonance meaning the frequency of interest is reflected.

**II.2. Effective Parameters**

These effective parameters outline the reaction of electromagnetic waves when moving through the now ‘continuous medium’ of the meta-material. Eq. (2) and (3) show the impact of these materials for the propagation of electromagnetic waves through this complex medium.

\[ \nabla \times E = -\frac{\partial}{\partial t} \mu_0 \mu_{\text{eff}} H \]

\[ \nabla \times H = \frac{\partial}{\partial t} \epsilon_0 \epsilon_{\text{eff}} E \]
Where $H$ is the magnetic field, $E$ is the electric field, $\epsilon_0$ is the electric permittivity in vacuum and $\mu_0$ is the magnetic permeability in vacuum. Due to the inter-change of power between the electric and magnetic field it’s clear that the effective values of $\mu_{\text{eff}}$ and $\epsilon_{\text{eff}}$ will control the propagation of the electromagnetic waves in the medium. Extracting the effective parameters from the designed material is done through the S-parameters. This resonance can be identified in the S-parameters by strong reflection of the power at that frequency of interest. There is also a significant phase change from positive to negative which leads to a change of $\epsilon$ and $\mu$ from their normal positive values and will eventually produce $\epsilon_{\text{eff}} < 0$ and/or $\mu_{\text{eff}} < 0$.\(^2\)

Smith et. al.\(^3\) proposed a method to calculate these effective parameters once the S-parameters are known, and since then there have been several robust equations that provide the effective parameters of the material. Using this rationale Menzel et. al.\(^4\) used Eqs. (4) and (5) to find the effective parameters of the meta-material:

\[
\begin{align*}
    n_{\text{eff}} &= \pm \frac{1}{kh} \cos^{-1}\left(\frac{1}{t} n_1(1 - r^2) + n_3 l^2 + r(n_3 - n_1)\right) + \frac{2\pi m}{kh} \\
    z_{\text{eff}} &= \pm \sqrt{\frac{(1 + r)^2 - t^2}{n_1^2(1 - r)^2 - n_3^2 t^2}}
\end{align*}
\]

Where $n_1$ and $n_3$ are the refractive indices of the superstratum and substrate respectively, $r$ is a normalized complex reflection coefficient taken at the beginning of the slab, $t$ is a normalized complex transmission coefficient taken at the end of the slab.

From these two parameters it is possible to obtain the effective permittivity and permeability

\[
\begin{align*}
    \epsilon_{\text{eff}} &= \sqrt{n_{\text{eff}} z_{\text{eff}}} \\
    \mu_{\text{eff}} &= \sqrt{n_{\text{eff}} z_{\text{eff}}}
\end{align*}
\]

which again shown by Menzel et. al.\(^4\) are quite often not very meaningful due to the strong spatial dispersion. For our structure examples of these parameters at a resonant frequency of 1THz can be found in Figure 4.

From this Figure we can see that close to the resonant frequency $\mu_{\text{eff}} < 0$ and $\epsilon_{\text{eff}}$ is mostly positive except in a very narrow region. Therefore it can be concluded that this structure has $\mu_{\text{eff}} < 0$ and $\epsilon_{\text{eff}} > 0$.\(^4\)
III. Tunable Filter

META-materials offer a unique way of designing a tunable filter, in which the SRR resonant frequency can be tuned if either the capacitance or inductance of the SRR are changed. The different methods to dynamically tune a meta-material are briefly explained below.

III.A. Methods of Tuning Metamaterials

III.A.1. Liquid Crystal

Liquid Crystals possess a large birefringence (property of having a refractive index that depends on the polarization and propagation direction of light) which can be controlled by an external electric field, leading to the control of the metamaterial’s resonant frequency. Experimentations have been conducted where the method works by having the SRR of the metamaterial unit cell immersed in a layer of Liquid Crystal whose properties are controlled by the orientation of the molecules in the liquid crystal.

III.A.2. Semiconducting Substrate

If we inject an electrical current through a semiconducting material, we change the refractive index of this material. If this semi-conductor is placed as a substrate of a meta-material structure and we change the electrical current flow into the semi-conductor then we shift the resonant frequency of the combined structure.

III.A.3. MEMs

Micro-electro-mechanical Systems (MEMS) are described as “the integration of mechanical elements, sensors, actuators, and electronics on a common silicon substrate through micro fabrication”. It is becoming a big area of research for the tuning of meta-materials as it offers a means of changing the orientation or structure of the unit cell for the meta-material lattice. There has been work conducted within the THz spectrum with regards to MEM based meta-materials. Especially with regard to out of plane movement of SRR’s, MEMS technology has developed the cantilever, which is designed to “pop-up” out of plane in response to a thermal stimulus which can be seen in Figure 5. This technology has made conceptual simulations of tunable resonances physically achievable with planar and out of plane tuning both being possible.

III.B. Motivation

The lack of commercial applications in the terahertz range is referred to as the terahertz gap, but with recent advances in higher power sources and more sensitive detectors, there is a growing range of potential uses for terahertz devices. The exploration of tunable filters within the terahertz range will open up a larger portion of the spectrum for use rather than the dimensions...
of the unit cell restricting it to the one frequency. Applications for terahertz devices include Material Characterization, THz imaging and Biomaterial THz Applications\textsuperscript{12} demonstrating that a tunable meta-material filter at terahertz frequencies will have a significant impact on defence and security.

III.C. Planar Array

Starting from a planar array of the SRR, different orientations were tested while moving the resonators across the unit cell in reference to each other as depicted in Figure 6 and 7. The polarisation of the incoming wave must have the electric component across the gap in the SRR with the magnetic component perpendicular to the base of the SRR. This ensured that both an inductive and capacitative reaction occurred for the resonance. We assume that the plane wave is perpendicular to the plane of the structures.

From Figure 9 it can be seen there is a maximum shift in resonances of approximately $50\text{GHz}$. The resonant frequency changes periodically with the traverse distance between 0.97 and 1.02 THz. This demonstrates the periodicity of the lattice structure shown as the SRR approaches the opposite end of the $160\mu m \times 160\mu m$ unit cell (traverse = $-70\mu m$), where the reaction with the SRR in the adjacent unit cell will be similar to that of the SRR within the unit cell at traverse = $30\mu m$. As the two SRR approach each other within the unit cell a larger resonance shift was seen, this can be attributed to the coupling of the two SRRs which caused the capacitance to increase between the two SRRs. The maximum shift can be seen in Figure 9.

Figure 7 shows a different orientation for the SRRs, which resulted in a wider range of resonances by an extra $20\text{GHz}$ ($\approx 70\text{GHz}$) to the setup in Figure 6. This may be attributed to the coupling between the inductances of the individual horseshoe structures leading to a shift in the resonant frequency. This is similar to what happens in a transformer where the inductance of one SRR affects the current flow in the other SRR. The maximum shift in transmission can be seen in Figure 6.

Planar movement within a meta-material lattice has been postulated and tested at the microwave level with mechanical shifting of the substrate at different levels of the meta-material lattice. Lapine et. al.\textsuperscript{13} conducted this experiment and proposed it to be effective at all frequency levels with the correct scaling of the SRR. An example of a MEM Comb Drive achieving a Planar shift can be seen in Figure 8.
The reference frequency refers to the position where the SRRs are perfectly aligned with no mechanical shift.

III.D. Out of Plane Array

After exploring the in plane movement of SRR’s, also explore out of plane displacement of the structure. Initially the polarisation of the electric field is always kept along the gap of the SRR. But we also studied the case in which the electric field will change its orientation warranting investigation into moving the SRR so as to completely shut off the resonance. In the following simulations the variable 'turn' refers to the angle of rotation with turn = 0 being in plane.
Orientations that would result in continued coupling throughout a change from planar to out of plane with a normal incidence were then looked at. Figures 11(a) and 11(b) show the two basic orientations used which would maintain a workable orientation throughout the shifting. It was predicted that only two positions would need to be simulated as these would offer the extremes within the resonance shifting also the strongest responses. The response to Figure 11(a) located at Figure 12 shows that when the SRR is moved out of plane the resonance splits into two resonances, as well as shifting to a higher frequency than the planar response. The splitting is because the symmetry is broken and the coupling of individual elements will result in the creation of a new resonant frequency (this is similar to what happens when two similar atoms are close to each other and the energy levels are split).

![Diagram](a)

![Diagram](b)

Figure 11.

Figure 13 shows the transmission response of Figure 11(b), and it can be seen that this response is much 'cleaner' with a single resonant frequency, but still offering a large resonance shift when compared to planar tuning. Comparing the orientation at Figure 11(a) with Figure 11(b), it can be hypothesised that the inductive and capacitative effect will change with each orientation with Figure 11(b) will have a stronger capacitative effect but weaker inductive effect, due to orientation with the quartz substrate but these effects will be opposite for Figure 11(a) due to it’s orientation with the quartz substrate.

![Graph](Graph)

Figure 12. Transmission of Figure11(a)
Looking at the concept of a switch allowing the transmission of the signal and no resonance after moving the SRR out of plane. Figure 14 shows the orientation tested for a switch and Figure 15 shows the resultant resonances. From this simulation it could be seen that the resonance got weaker the more the SRR moved out of plane, this could be ascertained by the reduced electric coupling as the capacitative component became weaker with no electric field across the gap. With the SRR perpendicular to the plane there was no longer any resonance as seen by Figure 15 because the electric field has the wrong polarisation.

IV. Experiments

The fabrication of the Horseshoe SRR involved a gold SRR on a quartz substrate, the SRR’s were bonded to the quartz substrate using a small titanium layer of $\approx 1\text{nm}$ and the overall thickness of the SRR is $200\text{nm}$. The SRR’s were structured in a $2\text{cm} \times 2\text{cm}$ array made up of $24\mu\text{m} \times 44\mu\text{m}$ unit cells. This design was not based on the design in Figure 2 but will validate the simulations conducted with planar arrays. Figure 16(b) shows the dimensions of the horseshoe.
Terahertz time-domain spectroscopy (THz-TDS) was used to obtain the signals transmission through the meta-material in the time domain. To achieve the transmission coefficient, first the strength of the EM wave had to be determined in free space, this measurement provides a reference. Then the Horseshoe array was placed within the path of the EM wave where the strength of the transmitted wave was then measured. From these measurements the Transmission was determined using MATLAB from the FFT of both the reference signal and SRR signal. The signal also went through a gaussian filter to remove Fabry-Perot resonances, which cause delayed spikes following initial transmission spikes. Fabry-Perot resonance are caused by internal reflection in the material before exiting the material later than the initial signal transmitted through.

From Figure 17 it can be seen that there is a slight shift from the resonance obtained in simulations to the resonance obtained from the THz-TDS result. This can be attributed to several factors involved with the simulation model. The titanium used to bond the SRR to the quartz in the physical experiment was not used in the CST simulations as the properties of this material at THz frequencies were not quantified. THz is also susceptible to interference from water vapour that can be present in the testing apparatus despite efforts to eradicate it’s presence. These factors will contribute to the strength of the resonance as well as a slight shift from the simulated transmission spectra. The shape of the experimental spectra still follows that of the simulated, therefore validating that design parameters in CST adequately simulates the reactions in the physical world.

V. Recommendations

The modelling conducted in this project was largely conceptual, with planar and out of plane simulations conducted with CST, the resulting resonance shifting proves that this area is worth further exploration in determining designs that give a ‘clean’ resonance shift over a large frequency band. The reason that a large band for resonance shifting is important is due to the flexibility that it gives to the THz band, and controlling the frequency to use.
The model used for simulating these designs could also be improved, moving from conceptual tuning designs, the latest publications released about MEMs will offer the ability to simulate the MEMs design in order to determine their suitability for use within a tunable filter. Research into how the model is tested physically and then attempting to design a model that simulates it with a high amount of accuracy will assist the design process in this area.

The tests conducted in this project will be very useful for the research conducted by Mr Liming Liu who is currently researching means of achieving tunable filters with meta-materials.

VI. Conclusion

In conclusion the research for this project focussed on the effective parameters of meta-materials and how it related to the resonant frequency of the meta-material. The main focus of the project was to look at methods of tuning the meta-material by shifting the resonant frequency, this focused on physical shifting of the SRR both planar and out of plane with regard to the unit cell. Through simulations, unit cells were constructed and tuned, shifting both planar and out of plane.

From these simulations it was determined that out of plane tuning resulted in a larger shift of up to 100 GHz where planar shifting only achieved around 60 GHz. The research that has been conducted into tunable meta-materials has offered the means to realise both the planar and out of planar tuning that has been simulated in this project. These tunable filters may find applications in terahertz sensing, detection of hazardous substances and terahertz switching.

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References


