Development of a background oriented schlieren system

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The background oriented schlieren (BOS) flow visualisation method is a relatively new method of determining flow characteristics. The technique offers a significant decrease to the cost and equipment requirements when compared to established flow visualisation techniques. BOS uses computational analysis of images to provide an insight into the features and processes occurring within a flow field. This project aimed to determine the optimal setup for the system, and to investigate the effect of varying contributing parameters within the system on the systems performance as a whole.

Contents

Nomenclature (examples – include units where appropriate)

B	=	position of background
F	=	position of flow field
L	=	position of camera lens
I	=	position of camera image plane
β	=	deflection angle
n	=	refractive index
ρ	=	density
G	=	Gladstone-Dale constant
f_{lens}	=	focal length of the static test lens
Δ_{LI}	=	distance between camera lens and image plane (focal length)

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Δ_{BF}	=	distance between flow field and background
Δ_{BL}	=	distance between camera lens and background

I. Introduction

Flow visualisation is an interesting and important component in the study of gas dynamics. It provides a means of determining flow characteristics and creating images of the processes that are occurring in an otherwise invisible medium. Flow visualisation techniques operate through detecting changes in refractive index within the medium, which alters the light that passes through it. These refractive index changes are caused by density changes within the flow field, and when quantified this information can reveal information about other flow variables including temperature, pressure and velocity. All of this is possible without direct measurement that would otherwise disturb the flow, giving accurate results that aren't affected by instrumentation procedures [1][2].

Three main techniques are used in flow visualisation: shadowgraph, schlieren and interferometry. These techniques rely upon measuring different changes caused by the refractive index change, shadowgraph measuring displacement of rays, schlieren measuring deflection and interferometry measuring the phase shift [1][3]. One factor that is common to all of these techniques is the requirement for complex, high quality optical equipment and skilled personnel to set up and perform the visualisation. This makes flow visualisation inaccessible for many groups that would otherwise benefit from the information it can provide. Restrictions on the equipment precision required for correct images also limit the possible environments in which the techniques are able to be utilised [4]. None of the techniques is inherently quantitative, and precise calibration is required to make them so. The size of the flow under investigation is also limited, as optical equipment size typically increases as the flow field under investigation gets bigger. Each of these limitations increases the cost, reduces the range of application and reduces the ability of unskilled personnel to access flow visualisation.

Background oriented schlieren is an effective alternative to these methods that requires very few, easily accessible components and very little expertise to perform. Whilst still in its infancy compared to the more established methods, the simplicity of the setup allows the method to obtain results that are comparable to those obtained from the other flow visualisation techniques [5] without the need for skilled personnel and expensive optical components. The main setup requires only a camera and a structured background. Two images are taken, one of the background and another of the background with the flow field between camera and background, and these are compared computationally to determine the light ray deflections that occur when the flow field is included in the image. The BOS method has fewer limitations on its field of view than other methods, those being that the background must be of a similar size to the flow field and that the camera must be sufficiently far away from the flow field [6][7]. This makes the method applicable in a wide range of applications, including visualisation of wing tip vortices on in flight rotor- and fixed wing aircraft, large scale wind tunnel tests and full-scale blast wave quantifications, along with most experiments currently utilising other visualisation methods [8].

This report will detail the methodology used in the optimisation of a background oriented schlieren system, and provide explanation of observations made within the experiment. It will also define the scope of future work that was unable to be achieved in the project.

II. Historical Efforts

Background oriented schlieren was developed primarily to alleviate the issues mentioned above in traditional schlieren systems. The first use of a system using BOS concepts was in the work of Köpf (1972) using laser speckle patterns to detect deflections in candle flames [9]. Without the use of modern computers, the images were compared by capturing a reference image and a disturbed image on the same photographic plate, revealing interference fringes similar to those seen in interferometry. With the advent of readily available, high speed computers, a similar technique to this has been used to great effect with very small error being achievable [10]. Whilst this is a very useful field of research, it still suffers from relying upon several optical components that require specialist personnel to operate.

To reduce the total required optical components to just a camera, Meier developed a system wherein the light field did not need to be artificially created [8]. Naming the technique "background oriented schlieren", Meier did not create a structured light field through the use of lasers, rather used a printed background of random noise to create a light field which showed deflections that could be measured easily. This method required only two components to conduct an interrogation of a flow field: a camera and a structured background

and all analysis of the images was through computational post processing. Further work in the field has drawn from particle imaging velocimetry and various tomography disciplines to increase the variety of situations that BOS is able to be used [2]. Significant work has been conducted in the areas of algorithm optimisation to increase the speed of the processes [11] as well as defining the range of possible setups through sensitivity analysis [2][7]. Each of these works used a single background and altered other parameters within the experiment to optimise the method.

III. Project Aims

The aim of the project was to conduct a systematic study on the influence of contributing parameters on the background oriented schlieren system, specifically the shape and size of dots or elements on the background. The core phases of the project were to develop a working background oriented schlieren system, optimise this program to increase sensitivity and reduce error, and to systematically vary the physical parameters within the system to discover their effect on the output. This report will aim to communicate and explain the results achieved, and also to demonstrate the applicability and value of BOS in the modern flow visualisation toolbox.

IV. Theory and Methodology

A. Setup and Concept

The setup used in background oriented schlieren is shown in Figure 1. Light emitted from the background (*B*) passes through the flow field (*F*) and the lens (*L*) ending at the camera image plane (*I*). Without the flow field to deflect it, a ray that is emitted from point *a* will transmit its information to point *e* on the image plane, and similarly the ray from point *b* will end at point *d*. However, when the light from the background moves through the flow field, the rays will be deflected, and the ray from point *a* will be deflected through angle β at point *c* and will be appear on the camera image plane at point *d*. Comparing the both the reference and flow field images will show different detail at the same position, indicating deflections in the light rays. Capturing an image of both the undisturbed background and the background through the flow field allows the use of computational post processing to compare the two images, allowing the operator to find the angle β through which the ray was deflected.

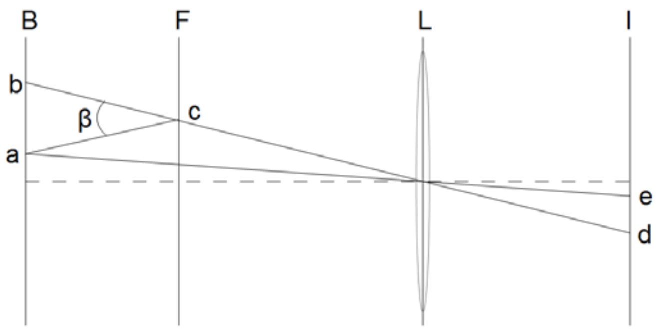

Figure 1. Simplified ray path diagram

For the above explanation to be true, two assumptions have been made about the nature of the flow field: that the deflection takes place in a single plane and is thus immediate, and that the change in the path of the ray is caused entirely by deflection; so no displacement is observed. These assumptions are valid when the thickness of the flow field is much smaller than the distance between the camera lens and the background.

To ensure that the background transmits information that has traceable deflections, a random noise pattern of dot elements is used. The elements of random noise ensure that when a small area of the background is sampled, there is a high probability that it will be unique in its local area. This uniqueness allows for the algorithm to determine the most likely displacement that occurs between the reference and flow field images at each position that is sampled. The backgrounds used in this project were generated using the Matlab rand function to define the coordinate positions of black shapes on a white background. As the background is the part of the system that carries the information, it is required to be the focus of the camera. For the greatest effectiveness, the flow field also needs to be in focus to allow the information it contains to be transmitted back to the camera for evaluation. This is achievable through use of a long focal length camera lens and small aperture to increase the depth of field, as well as ensuring that the flow field is close to the background.

With the information of the angle β, one is able to find the refractive index gradient using Eq. (1), where $\partial n/\partial y$ is the refractive index gradient of the examined point in the flow field. Equation 2 is then used to convert the refractive index gradient into density gradient to resolve the flow field into the relevant units, where G is the Gladstone-Dale constant and can be found in most chemistry handbooks.

$$\beta = \frac{1}{n_0}\int \frac{\partial n}{\partial y}\partial z \qquad (1)$$

$$n - 1 = G \times \rho \qquad (2)$$

B. Sensitivity

Sensitivity is defined as the ratio of the detected change (the distance between points *e* and *d*) to the change in the parameter being measured (β). Some geometric manipulation and application of the thin lens relation allows the following expression to be defined:

$$S = \frac{\Delta_{ed}}{\beta} = \frac{\Delta_{LI}\Delta_{BF}}{\Delta_{BL}} \tag{3}$$

Where Δ_{ed} is the distance between points *e* and *d*, Δ_{LI} is the distance between the lens and the camera image plane (can be estimated as the camera focal length), Δ_{BF} is the distance between the background and the flow field, and Δ_{BL} is the distance between the background and the camera lens. The full derivation can be found in Ref. 7. This relationship allows for the optimisation of the system's geometric parameters to ensure that the sensitivity is high enough to fully resolve the smallest expected deflection for the given camera resolution.

Equation 3 shows that the sensitivity of a BOS setup is able to be increased through several different methods: increasing the distance between the background and the flow field, increasing the camera resolution or increasing the focal length of the camera. Increasing the distance between the background and the flow field will bring the flow field out of focus, making it a less ideal parameter to vary, however the maximum distance between flow field and background should be reached whilst the flow field is still in focus. The camera resolution is not a very limiting factor for steady flows as DSLR and other high resolution cameras may be used. However, for the study of unsteady supersonic flows which require the use of high speed cameras, the resolution is limited, and careful consideration should be made about the sensitivity before setting up an experiment. Increasing the focal length of the camera also increases the magnification, allowing the camera to be placed a greater distance from the background, whilst also increasing the depth of field in which the flow field is able to be placed. A large focal length is recommended for all BOS applications, however equipment and laboratory size constraints may cause issue with this requirement.

C. Background Element Shape

The evaluation program used to determine the vector field is sensitive to the edges of the background elements used, rather than the areas. With more edges within each interrogation window, the window becomes more unique, and thus has less probability to become an outlier. Two different background element shapes were examined due to their ease of generation: squares and circles. An example of each background is shown in Figure 2. For both backgrounds equal areas of black and white were used to ensure comparison was made directly between the length of the edges of each different element type. Using the equations for both perimeter and area of squares and circles, and equating the two areas it is easy to see that square elements will have a longer perimeter length by a factor of $4/\pi$, reducing the chance of an outlier appearing by the same margin. It is predicted that the backgrounds utilising square elements should have a decrease in the percentage of outliers by this factor.

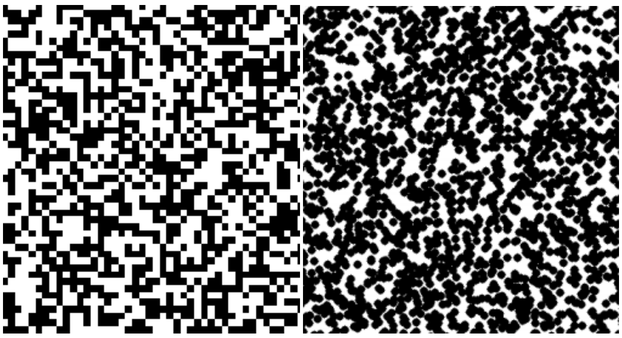

Figure 2. Square vs round elements

D. Experiment

To test the systems response to different background types, a repeatable static test was designed using a long focal length lens. Usage of a lens has the advantage of having a set range of deflections that provide similar results for all backgrounds placed behind it, allowing for the examination of the effects of changing backgrounds without having to account for varying effects from the flow field. The camera used was a Canon EOS10D and was aligned to the centre of the lens. The backgrounds were interchanged without any disturbance to the setup, and several reference images were taken both at the start and end of each run to ensure any disturbances between images could be mitigated through comparison to a different reference image. Illumination of the background was provided by a large floodlight which provided a steady light source that eliminated any error caused by flickering light sources such as fluorescent tubes.

V. Computation Procedure

A. MatPIV

The evaluation software used was the MatPIV toolbox commonly used in particle imaging velocimetry. Some modification to the code was required to ensure that the outputs were usable in this application, but the underlying purpose of the code is the same as what is required for BOS. The program works by taking an interrogation window from the background and comparing its contents to nearby areas on the flow field images, assigning them a correlation score based on how similar they are to the original image. Subpixel accuracy is achieved by taking the correlation scores near to the maximum and fitting them to a curve to find the point at which the actual maximum would lie. The program then generates a vector from the centre of the window on the background image to its calculated position on the flow field image. This process is repeated for all positions on the background image to generate a vector field that shows the relative image shift at each point. Two parameters control this process: the size of the interrogation windows and the overlap of each window on the background and the effect of altering each of these is discussed in the results section.

B. Filtering

Due to the vector field being based off the correlation between different window images, false positives are frequently created and are required to be filtered out to ensure that the image contains the least error possible. These outliers tend to be very large in size and so are easily eliminated through a filter that compares each vector to the mean of the vectors surrounding it. This process is very computationally expensive, as several vector operations are required for each element in the vector field. Another disadvantage of this method of filtering is that it causes degradation of sudden or step flow features, such as shockwaves.

The setup is also prone to uniform noise as the captured images are taken seconds and even minutes apart, allowing time for the camera setup to be moved through vibration or just to be bumped by an operator. Any lateral displacement of the camera will present itself in the vector field through all of the vectors having a shift in the direction of the change. Whilst this error should be mitigated by rapid image capture and careful control of the camera system, it can be corrected for computationally through the use of a uniform noise filter. The filter used in this analysis was the only filter that changed the values of the vectors within the flow field rather than eliminating those that were outliers. This creates an opportunity for computer generated error to enter into the results, however the extent of the uniform noise meant that the use of such a filter was necessary, and any computer induced error would be far less than the error caused by the uniform noise.

C. Error Analysis

As the experiments were conducted using a lens as the flow field, a computational model was able to be created to define the deflections that the lens would generate, and what the BOS method should evaluate. It was assumed that the rays transmitted from the flow field to the camera would be parallel, an assumption that holds for the relatively large distance between camera and flow field. For a convex parabolic lens, all light rays would then originate from the focal point *F*, as seen in Figure 3. Similar triangle laws are then used to obtain the simple relationship of $\Delta p = y * d_{BG}/f_{lens}$. The two dimensional version of this is used to create a computational model of the deflections expected from the lens. The error is then defined as the normalised difference between the experimental result and the theoretical result. Normalising the differences was required to achieve non-dimensional results, however the very small deflections near the centre of the lens led to high error being calculated there. To mitigate this, the average error was taken for each lens, smoothing the abrupt error spike at the lens centre, however the average error is much larger than the error seen across much of the rest of the image in most cases, and a much smaller error percentage is found at the areas closer to the edge of the lens. Whilst the error percentage was higher than what is seen over most of the image, the images are still comparable to determine which images contain the least error.

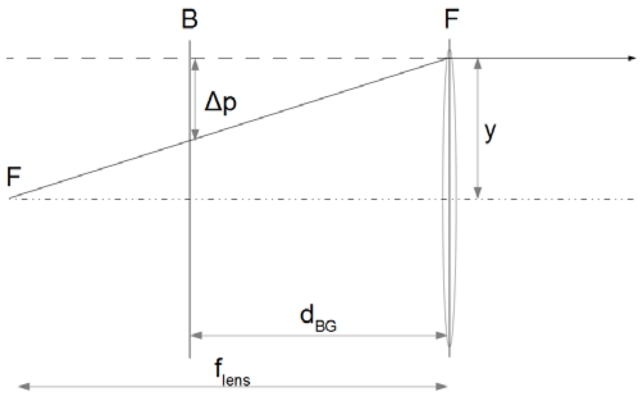

Figure 3. Lens model

VI. Results

Three factors were analysed in the capture of results in this project: error (described above), resolution and information loss. Resolution indicates the number of vectors that are output by the MatPIV function that are available to provide information on the deflections in the flow field. Information loss is the amount of noise that is removed through the necessary deletion of outlying vectors in the vector field. Error more fully describes the accuracy of a system than the other factors, however resolution shows the amount of detail that the system is able to achieve, and information loss provides a measure of the noise in the system, both of which are relevant from a practical perspective. All three factors are important to the investigation and need to be optimised for to achieve the highest quality result.

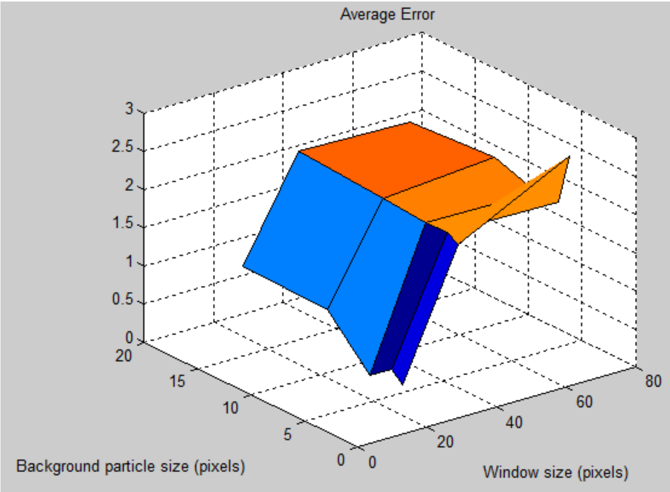

Figure 4. Error by window size – square background with constant overlap

A. Interrogation Window Size and Overlap

The interrogation window size was only able to be varied through three values due to limitations within the program. Whilst this limited the data points available for the investigation, a trend was easily identified in the results. Window sizes of 64 and 32 pixels were seen to have very similar error scores, but reducing the window size to 16 pixels reduced the error by 10-50%. The minimum error that was achieved with a window size of 16 pixels was 25%.

Decreasing interrogation window size increased the number of vectors generated by the program, increasing the resolution that was achieved with each. This increases the density of information that is able to be achieved on each image, and allows for a more detailed examination of the flow features. Computation time is also vastly increased, by a factor of 4 when halving the window size.

The loss of information remained fairly constant for changing window sizes, with a slight trend to decreasing loss for decreasing window size. This was surprising, as smaller interrogation window sizes contain less information, and so the uniqueness of the window decreases with its size, making the appearance of a similar part of the background nearby more likely. A high correlation score would be allocated to a nearby area that looks similar to the original interrogation window, causing the creation of an outlier. Filters would then eliminate this outlier, leading to a loss of information. With a higher probability of a nearby area having the same features as the local interrogation window, there should be a higher percentage of information loss, and the absence of this is puzzling.

The only reasonable explanation is that the effect is from the small sample size that was used in the experiment. The lens used was of a small diameter, meaning that probabilistic effects would not have a large amount of control over the effective features within the system. With only a small area of the background being used, the probability of repeating the pattern is diminished, even when viewed through a small interrogation window.

Overlap was not seen to change the error, as none of the calculations were computed with any different parameters. Changing the overlap parameter did vastly increase the number of vectors generated by the program, increasing the information density that was available. The information loss decreased for increasing overlap, as each of the extra vectors that were created were at a slightly different position than those created using a smaller overlap. Shifting the window a small amount keeps a lot of the information from the last window, meaning that any similar nearby regions will still have a high correlation score, and still possibly create outliers. Whilst this would create a larger number of outliers alone, the number of correct elements increases to a much larger extent, making the overall percentage of outliers decrease.

B. Filter Setup

During the experiment the effect of the different filters was examined in the hopes that a specific configuration of threshold values would be more useful for all images. This was not achieved because each image was found to require a different setup of filters to reach minimum error, indicating that the filters did not have an optimal configuration and that each picture requires separate treatment to reach the optimal configuration. The significance of this is in its implication for the computation requirements necessary to achieve a low error image. Iteration through all levels of filter are necessary to ensure that the setup that produces the lowest error is used, and that this setup does not remove too large a portion of outliers.

Whilst there was no set rule that could be applied to all images examined, decreasing the threshold that each filter used to define a passable vector did lead to results containing less error in most of the images. This is not a general rule however, as an optimum value for each filter was able to be found for each image. Deviating from this optimum value in either direction was found to increase the error in the image, as seen in Figure 5. This decrease in error is offset by a large increase in information loss as each filter also deleted many vectors that were not outliers.

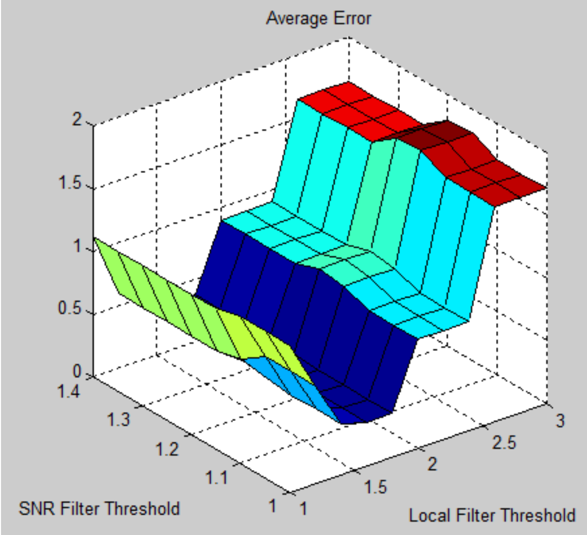

Figure 5. Error through filter variation – round background

C. Background Element Shape

Comparison between the different background shapes showed that the square elements did have a slightly lower information loss on average than the square elements, as seen in Figure 6. The reason for this occurrence is discussed in the Theory and Methodology section. The graph shown in Figure 6 shows that square elements have a sharp increase for larger background element size whereas information loss for round elements decreases. This is due to the fact that the window size becomes comparable to the size of the background particles. As the window size becomes close to the size of the background elements, many of the interrogation windows from the reference image will only contain one edge of an element, and will attempt to match this edge to others in the vicinity on the flow field image. This narrow field of view will cause only the refractive index gradients perpendicular to the surface to appear, as any deflection parallel will not cause any direct change in the image. Background elements that are smaller than the interrogation window will not suffer from this problem as their apparent motion is captured through the whole particle moving, whereas the tracking of a single edge does not allow for motion parallel to it to be tracked.

Square background elements will be more affected by this phenomenon, as each element only has two different directions in which perpendicular information can be transmitted. Any deflection information that is not perpendicular to the edge of the element will be lost, a total of 50% for each edge. This amount is not seen in practice as many of the interrogation windows will not contain just an edge, rather will capture a corner or the edges of several elements, for which there is less chance of creating an outlier. Using round elements decreases the chance of this happening, as even interrogation windows that contain just an edge are able to describe deflection in any direction due to the curve of the edge.

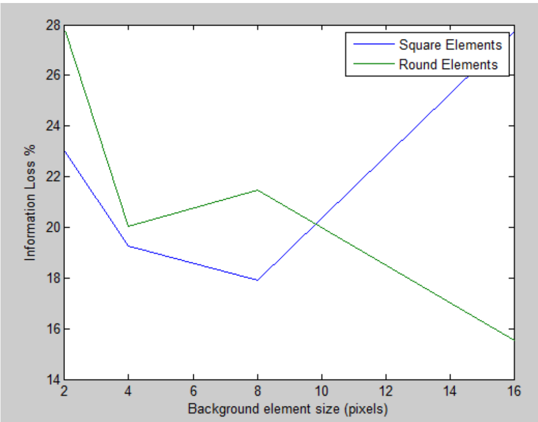

Figure 6. Information loss for different shaped elements

For the reasons described above, when the interrogation window must be of a similar size as the elements within the background, round background elements should be used, as they produce less outliers and have a slightly smaller error than square elements.

Some thought was given to using more complex, irregular shapes (such as stars or asterisks), with the intent of making the each interrogation window more unique. This would be an effective way of decreasing the number of outliers in the system, however is impractical in application. Each of the interrogation windows requires that all of its contents can be correctly visualised when captured by the camera. This means that the background shapes must be large enough that they are able to be fully resolved by the camera. For irregular shapes, the camera resolution must be large enough that the fine details of the background elements are distinguishable. If

the resolution isn't sufficient to show all of the detail in the reference image, different details can be revealed when deflected through the flow field. Attempting to find the shift between the two images through correlation becomes impossible as the interrogation window sees two different patterns between the reference and flow field images.

D. Background Element Size

The size of the background elements was chosen through matching the diameter of a particle to the number of pixels it would occupy on the camera chip. The chosen background element sizes were 1, 2, 4, 8 and 16 pixels, and they were chosen as they are multiples of the interrogation window sizes that would be used in the analysis, in the hopes they would identify a relationship between the interrogation window size and the background element size.

Analysis of the results showed very little difference in the amount of error produced between different background element sizes however a weak trend of decreasing error for smaller background elements was seen (Fig. 7). This trend did not extend to the smallest background size, being 1 camera pixel wide. The images generated by the camera for the smallest element background did not have clear definition between each of the individual background elements. This was further exacerbated when the flow field was included, as the changes in the image due to deflection also changed how the camera resolved that area, causing many outliers to form, vastly increasing the error and information loss at each point. This is similar to the expected behaviour of irregular shaped background elements, and should be avoided to prevent such poor information transmission.

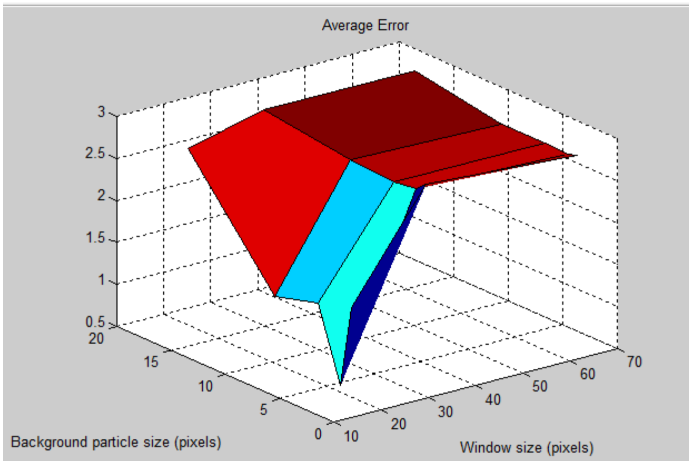

Figure 7. Error for changing background element size

Decreasing the background element size increased the percentage of elements that were eliminated through filtering (Fig. 6). This is against what was predicted, as one would assume that the smaller background elements would provide more detail within each interrogation window and thus be less likely to provide a false positive when compared to nearby patterns. This phenomenon is probably due again to the interrogation windows having difficulty in resolving background elements that are only a few pixels across, and that deflection through the flow field changes the way that the features within the window are produced by the camera imaging chip. Correlation relies upon the features within the interrogation window being shifted only in space rather than changing completely, and thus more outliers would be created when the contents of each interrogation window is not properly resolved by the camera imaging chip.

VII. Comparison with Existing Techniques

Background oriented schlieren has not been taken up as one of the more commonly used of flow visualisation, possibly because of the comfort of the industry in the older, more developed methods. BOS is a viable alternative to these methods, as can be seen in Figure 8, which shows the flow features that are visible using the BOS method compared to a shearing schlieren image. Most features within the flow are visible in the BOS method, and all are able to be quantitatively analysed to determine the density change across them. Some of the weaker regions have not been resolved in the BOS image, however these images were from different experiments and so may have subtly different results. The method does suffer in its resolution of strong shocks as a lot of detail can be seen to have been filtered out of the bow shock, due to the sudden strong change being interpreted as outliers or noise by the various filters.

Background oriented schlieren is able to be easily performed with rudimentary setups and limited equipment. All other flow visualisation techniques require highly precise optical equipment with trained personnel to set up and operate it, whereas the BOS image in Figure 8 was created with a digital SLR camera and an A4 sheet of printed paper. Any refractive index change can be viewed with this technique, and any person with a computer, a camera and a printer is able to achieve similar results as those shown.

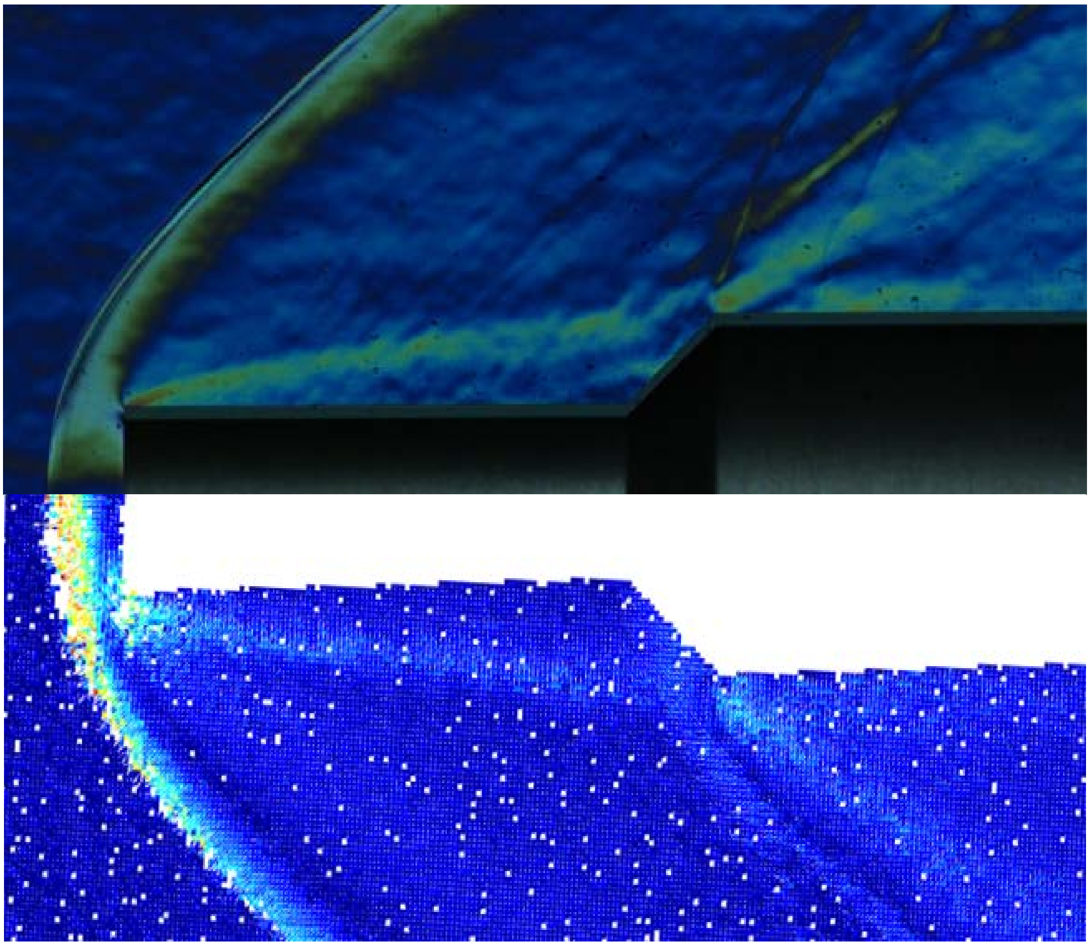

Figure 8. Shearing Schlieren above, Background Oriented Schlieren below

VIII. Conclusions

The aim of this project was to determine the effect of changing the parameters within a background oriented schlieren system, specifically the background pattern. The underlying physics that governs the deflection was investigated, and several parameters were determined that determine the success of the method. The optimal setup within the sample space was determined, and is described below.

Several constraints were used in this analysis that must be adhered to for the system to remain accurate. These constraints are:

- The distance between the camera and background must be much larger than the thickness of the flow field,
- A long focal length camera should be used
- The camera should be set at a small aperture.

The sensitivity of the system is a critical feature and is controlled by Eq. 3. High sensitivity should be attempted to be achieved, which is generated through increasing the distance between the background and flow field, increasing the focal length of the camera and increasing the camera resolution. The higher the sensitivity, the smaller the deflections in the flow field that can be visualised.

Through the use of the MatPIV program, it was seen that the lowest error was achieved through the use of small interrogation windows. Smaller windows decreased the error and decreased the information loss, as well as increasing the resolution of the vector field. Increasing the overlap vastly increased the resolution whilst also lowering the percentage of information loss. Increasing the resolution with these two methods also increased the computation times substantially. The filter method required for each image varied substantially, and each required iteration through the various types to determine the optimal setup that gave minimum error and minimum loss of information.

The background setup that yielded the least error and required the least information loss was contained square elements that were around two camera pixels wide. Smaller background elements should be avoided as the camera is unable to resolve them correctly and a large number of outliers is produced.

IX. Recommendations

Future work in this area should be focussed on a more quantitative analysis of the results obtained, with more experiments being conducted to ensure that all results follow a clear trend. Other parameters should be altered to determine their influence on the system, such as geometric layout of the components, differing lenses and a wider range of background shapes. These experiments were relatively easy to set up and perform, and a larger sample space would provide higher confidence in the results obtained.

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